A Note on Optimal Algorithms for Fixed Points

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22nd February 2010

Abstract

We present a constructive lemma that we believe will make possible the design of nearly optimal $O(d \log \frac{1}{\epsilon})$ cost algorithms for computing e-residual approximations to the fixed points of d-dimensional nonexpansive mappings with respect to the infinity norm. This lemma is a generalization of a two-dimensional result that we proved in [1].

1 Introduction

In [1, 2] we presented two-dimensional optimal complexity algorithms for computing residual ϵ-approximations to the fixed points of non-expansive mappings with respect to the infinity norm. These algorithms are based on bisection-envelope constructions and are derived from Theorem 3.1 of [1]. This theorem makes possible construction of a sequence of rectangles that contain fixed points and converge to the residual ϵ-approximation of some fixed point. At every iteration of the process the previous rectangle is cut by a factor of at least two, to obtain a new rectangle containing a fixed point.

In this paper we generalize the constructive theorem to an arbitrary number of dimensions $d \geq 3$, however, we are unable to utilize this new result in the construction of optimal algorithms.

The main obstacle in such construction is the ability to bound a new set containing fixed points by an "easy-to-construct" convex set of smaller volume and similar topological features to the previous set in this process. We stress that the two-dimensional sets in the optimal algorithm are rotated rectangles. What would be the proper sets in an arbitrary number of dimensions that would bound the non-convex sets resulting from the application of our general d-dimensional lemma?

2 Problem formulation

Given dimension $d \geq 2$, we define $D = [0, 1]^d$and the class F of functions, $f : D \to D$, that are Lipschitz continuous with constant 1 with respect to the