Some Explorations in SAT

Thomas C. Henderson, Amar Mitiche, Xiuyi Fan, and David Sacharny
University of Utah

UUCS-21-016

School of Computing
University of Utah
Salt Lake City, UT 84112 USA

14 July 2021

Abstract

We look at the SAT problem geometrically: for an n-variable problem, each corner of the hypercube is a possible SAT solution, and the interior points are possible Probabilistic SAT solutions. Given a Conjunctive Normal Form (CNF) sentence, $S = C_1 \wedge C_2 \wedge \ldots \wedge C_m$, over a set of Boolean variables $A = \{a_1, a_2, ..., a_n\}$, each conjunct, C_i, has at least one truth assignment that makes it false. If there are k literals in the conjunct, then there are 2^{n-k} truth assignments that make it false. A hyperplane can then be found for each conjunct which separates the solutions from the non-solutions. The intersection of the solution side of the hyperplane with the hypercube (the initial feasible region) produces a convex feasible region. Continuing this process for each conjunct, the result is a convex feasible region which may or may not contain a corner. Linear programming using the interior point method can be applied along each dimension to find the solutions with minimum and maximum values in that dimension. If neither of these has value 0 or 1, then there is no SAT solution. Moreover, our conjecture is that the interior point method can be made to find a corner solution if it exists. This is called the *Chop SAT* method. We also consider two other approaches which may prove useful in analyzing SAT: (1) count the number of solutions ruled out by each conjunct (this entails determining the number of new solutions eliminated by each conjunct), and (2) perform the interior point search in a non-Euclidean geometry (e.g., hyperbolic space). Current results are given for each approach.