250P: Computer Systems Architecture

Lecture 2: Basics of Digital Design

Anton Burtsev
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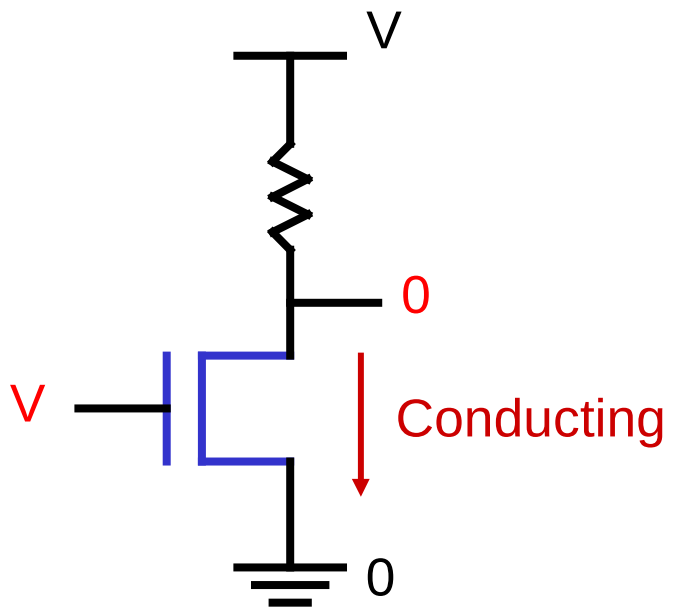
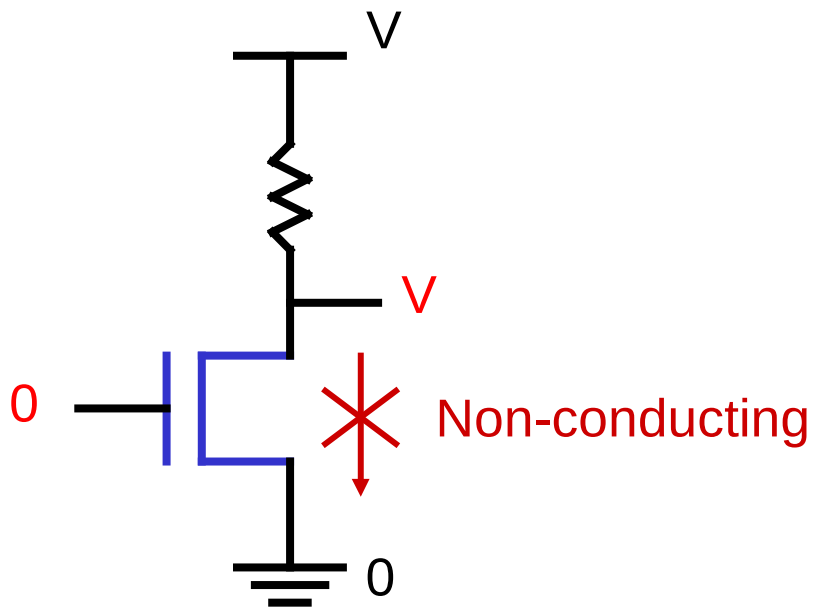
Digital Design Basics

- Two voltage levels – high and low (1 and 0, true and false)
  Hence, the use of binary arithmetic/logic in all computers

- A transistor is a 3-terminal device that acts as a switch





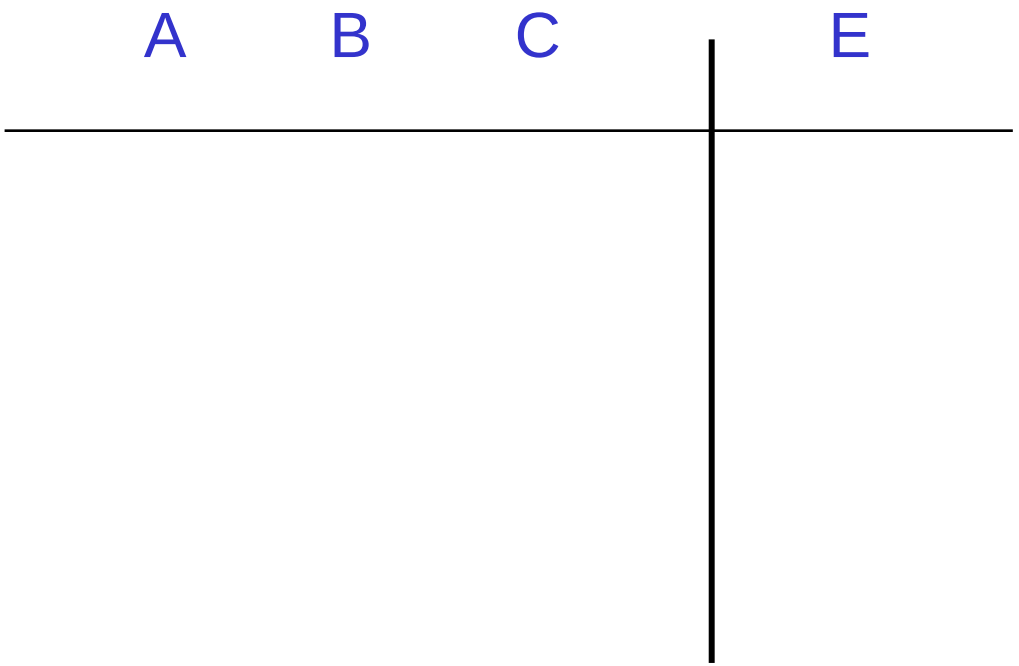
Logic Blocks

- A logic block has a number of binary inputs and produces a number of binary outputs – the simplest logic block is composed of a few transistors
- A logic block is termed *combinational* if the output is only a function of the inputs
- A logic block is termed *sequential* if the block has some internal memory (state) that also influences the output
- A basic logic block is termed a *gate* (AND, OR, NOT, etc.)

We will only deal with combinational circuits today

Truth Table

- A truth table defines the outputs of a logic block for each set of inputs
- Consider a block with 3 inputs A, B, C and an output E that is true only if *exactly* 2 inputs are true



Truth Table

- A truth table defines the outputs of a logic block for each set of inputs
- Consider a block with 3 inputs A, B, C and an output E that is true only if *exactly* 2 inputs are true

A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Can be compressed by only representing cases that have an output of 1

Boolean Algebra

- Equations involving two values and three primary operators:
  - OR : symbol + , X = A + B ➔ X is true if at least one of A or B is true
  - AND : symbol . , X = A . B ➔ X is true if both A and B are true
  - NOT : symbol $\overline{\ }$, X = $\overline{A}$ ➔ X is the inverted value of A

Boolean Algebra Rules

- Identity law : $A + 0 = A \quad ; \quad A \cdot 1 = A$
- Zero and One laws : $A + 1 = 1 \; ; \; A \cdot 0 = 0$
- Inverse laws : $A \cdot \overline{A} = 0 \; ; \; A + \overline{A} = 1$
- Commutative laws : $A + B = B + A \quad ; \quad A \cdot B = B \cdot A$
- Associative laws : $A + (B + C) = (A + B) + C$
  $A \cdot (B \cdot C) = (A \cdot B) \cdot C$
- Distributive laws : $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$
  $A + (B \cdot C) = (A + B) \cdot (A + C)$

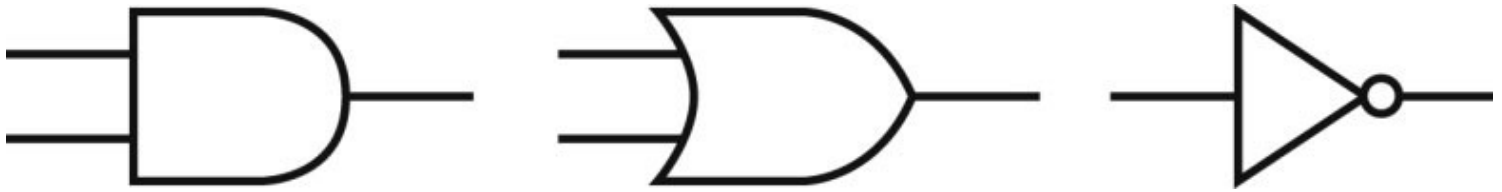
DeMorgan's Laws

- $\overline{A + B} = \overline{A} \cdot \overline{B}$
- $\overline{A \cdot B} = \overline{A} + \overline{B}$
- Confirm that these are indeed true

Logic for common arithmetic operations
Simple ALU

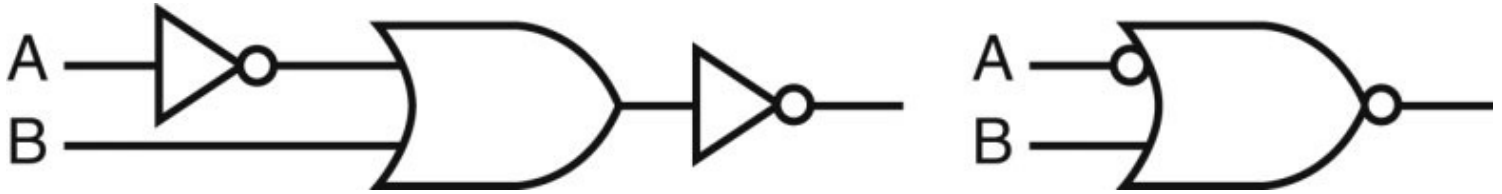
Pictorial Representations

AND OR NOT



Source: H&P textbook

What logic function is this?



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Boolean Equation

- Consider the logic block that has an output E that is true only if exactly two of the three inputs A, B, C are true

Multiple correct equations:

Two must be true, but all three cannot be true:

$$E = ((A \cdot B) + (B \cdot C) + (A \cdot C)) \cdot \overline{(A \cdot B \cdot C)}$$

Identify the three cases where it is true:

$$E = (A \cdot B \cdot \overline{C}) + (A \cdot C \cdot \overline{B}) + (C \cdot B \cdot \overline{A})$$

Sum of Products

- Can represent any logic block with the AND, OR, NOT operators
  - Draw the truth table
  - For each true output, represent the corresponding inputs as a product
  - The final equation is a sum of these products

A	B	C	E
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$$(A \cdot B \cdot \overline{C}) + (A \cdot C \cdot \overline{B}) + (C \cdot B \cdot \overline{A})$$

- Can also use “product of sums”
- Any equation can be implemented with an array of ANDs, followed by an array of ORs

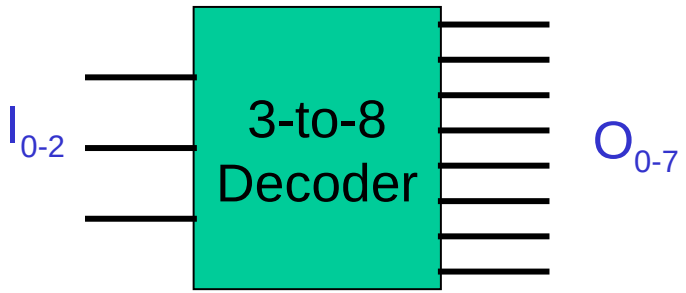
NAND and NOR

- NAND : NOT of AND : A nand B = $\overline{A \cdot B}$
- NOR : NOT of OR : A nor B = $\overline{A + B}$
- NAND and NOR are *universal gates*, i.e., they can be used to construct any complex logical function

Common Logic Blocks – Decoder

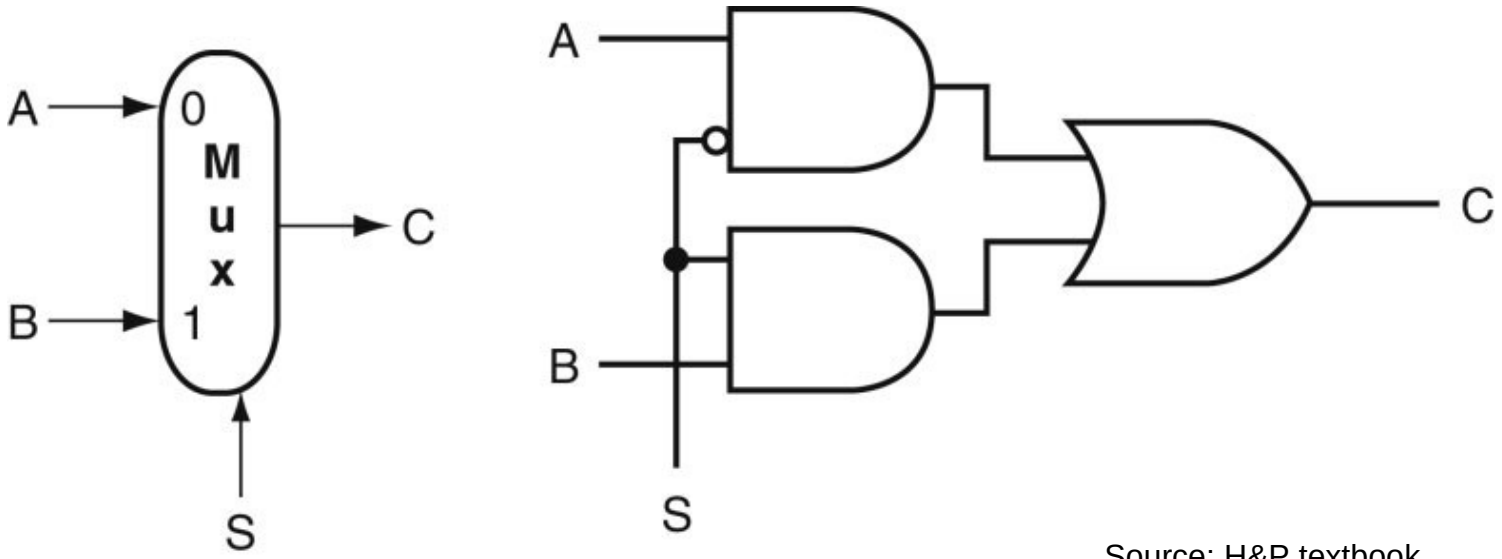
Takes in N inputs and activates one of 2^N outputs

I_0	I_1	I_2	O_0	O_1	O_2	O_3	O_4	O_5	O_6	O_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1



Common Logic Blocks – Multiplexor

- Multiplexor or selector: one of N inputs is reflected on the output depending on the value of the $\log_2 N$ selector bits



2-input mux

Source: H&P textbook

Adder Algorithm

	1	0	0	1
	0	1	0	1
Sum	1	1	1	0
Carry	0	0	0	1

Truth Table for the above operations:

A	B	Cin	Sum	Cout
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

Adder Algorithm

	1	0	0	1
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Truth Table for the above operations:

A	B	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

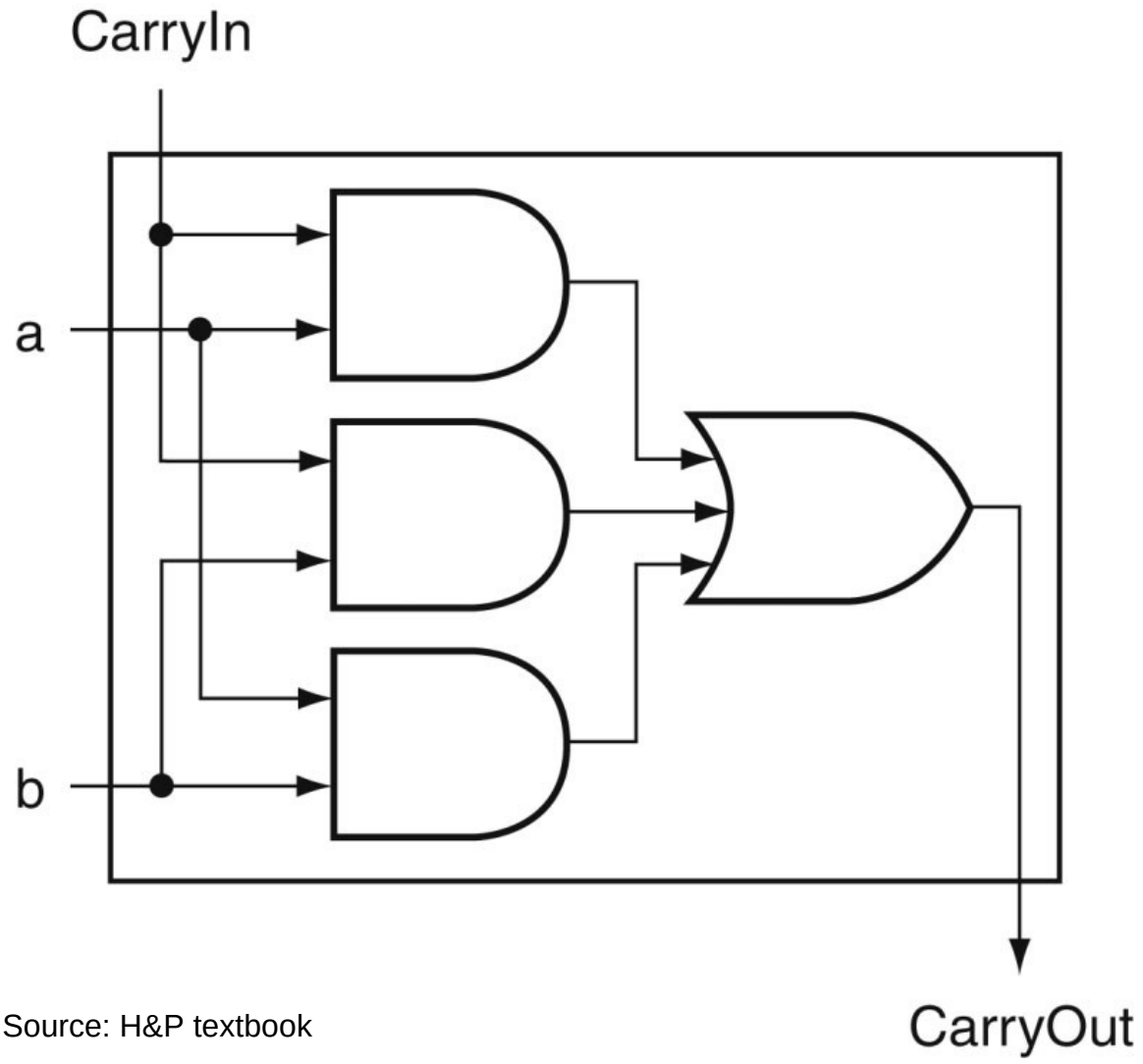
Equations:

$$\text{Sum} = Cin \cdot \overline{A} \cdot \overline{B} + B \cdot \overline{Cin} \cdot \overline{A} + A \cdot \overline{Cin} \cdot \overline{B} + A \cdot B \cdot Cin$$

$$\text{Cout} = A \cdot B \cdot \overline{Cin} + A \cdot B \cdot Cin + A \cdot Cin \cdot \overline{B} + B \cdot Cin \cdot \overline{A}$$

$$= A \cdot B + A \cdot Cin + B \cdot Cin$$

Carry Out Logic



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Equations:

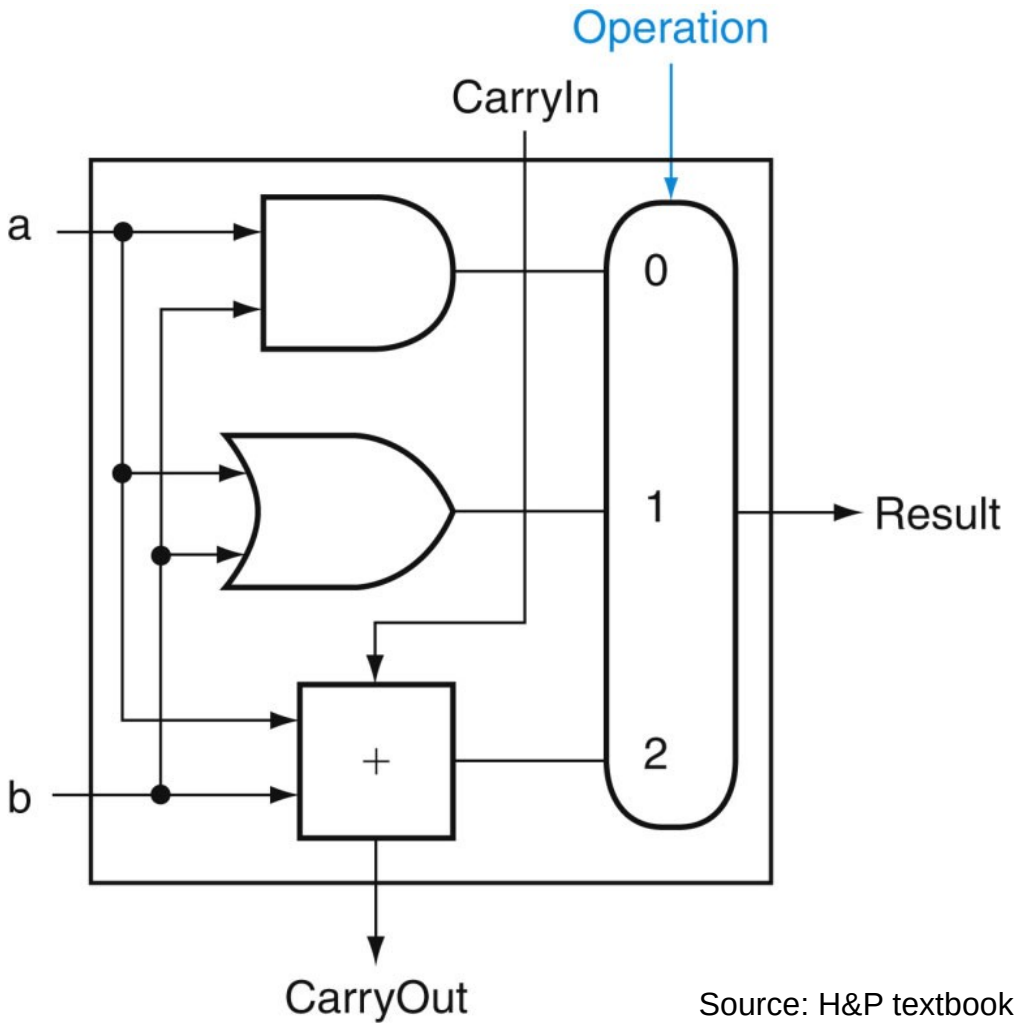
$$\text{Sum} = Cin \cdot \overline{A} \cdot \overline{B} + B \cdot \overline{Cin} \cdot \overline{A} + A \cdot \overline{Cin} \cdot \overline{B} + A \cdot B \cdot Cin$$

$$\text{Cout} = A \cdot B \cdot Cin + A \cdot B \cdot \overline{Cin} + A \cdot Cin \cdot \overline{B} + B \cdot Cin \cdot \overline{A}$$

$$= A \cdot B + A \cdot Cin + B \cdot Cin$$

1-Bit ALU with Add, Or, And

- Multiplexor selects between Add, Or, And operations



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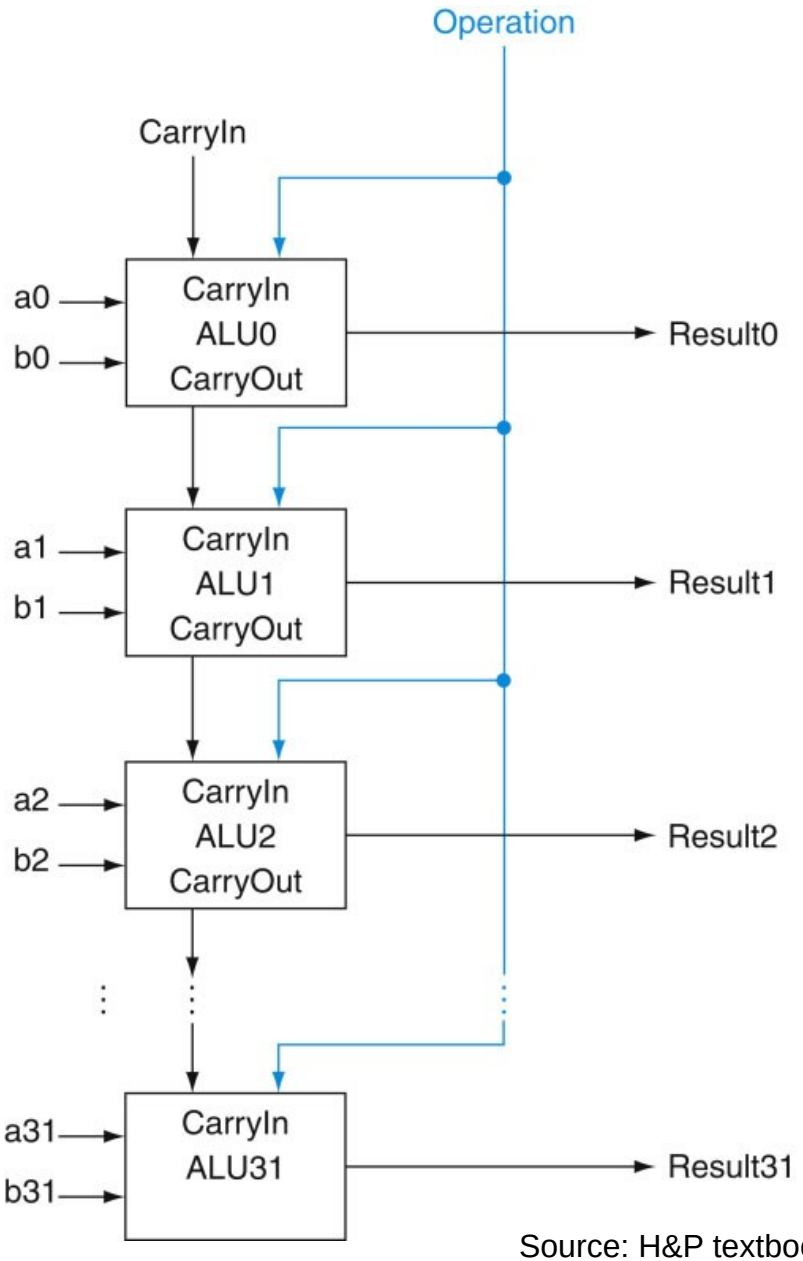
32-bit Ripple Carry Adder

1-bit ALUs are connected "in series" with the carry-out of 1 box going into the carry-in of the next box



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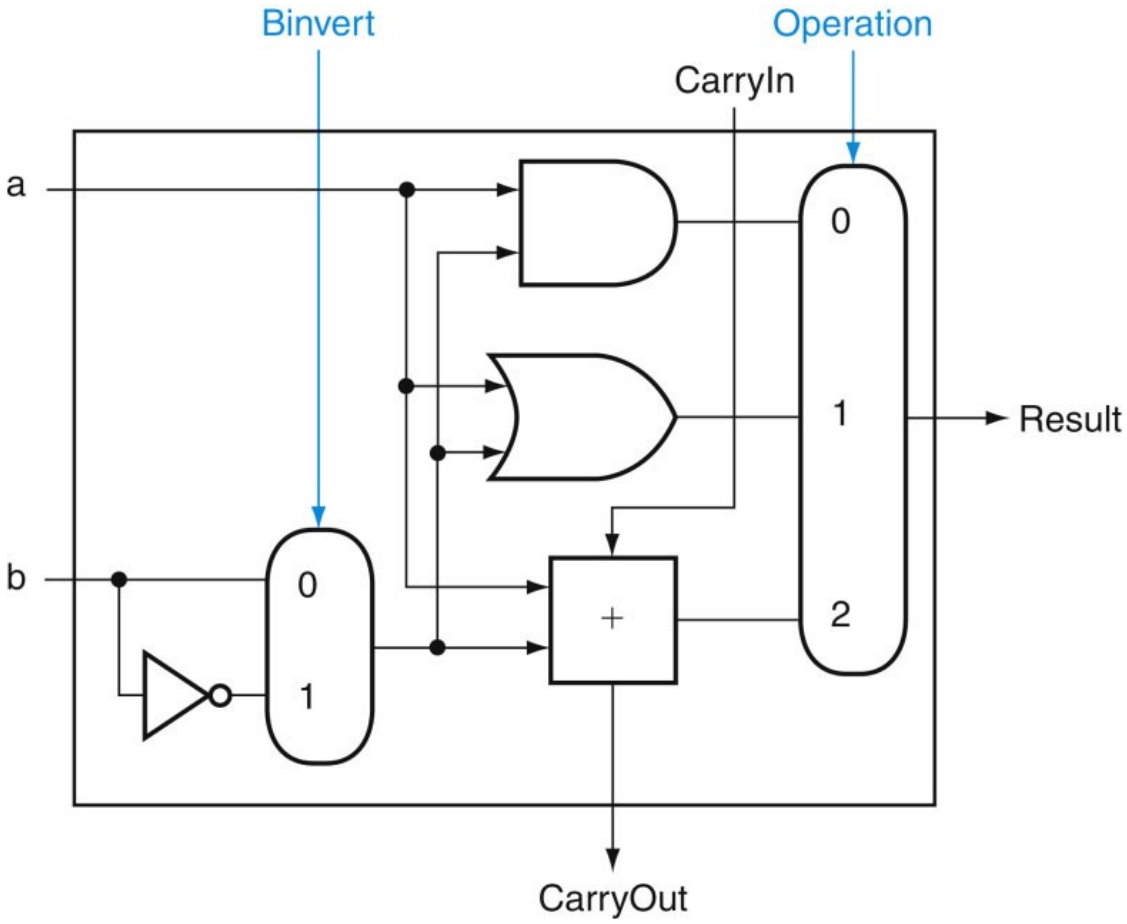
Incorporating Subtraction

Must invert bits of B and add a 1

- Include an inverter
- CarryIn for the first bit is 1
- The CarryIn signal (for the first bit) can be the same as the Binvert signal



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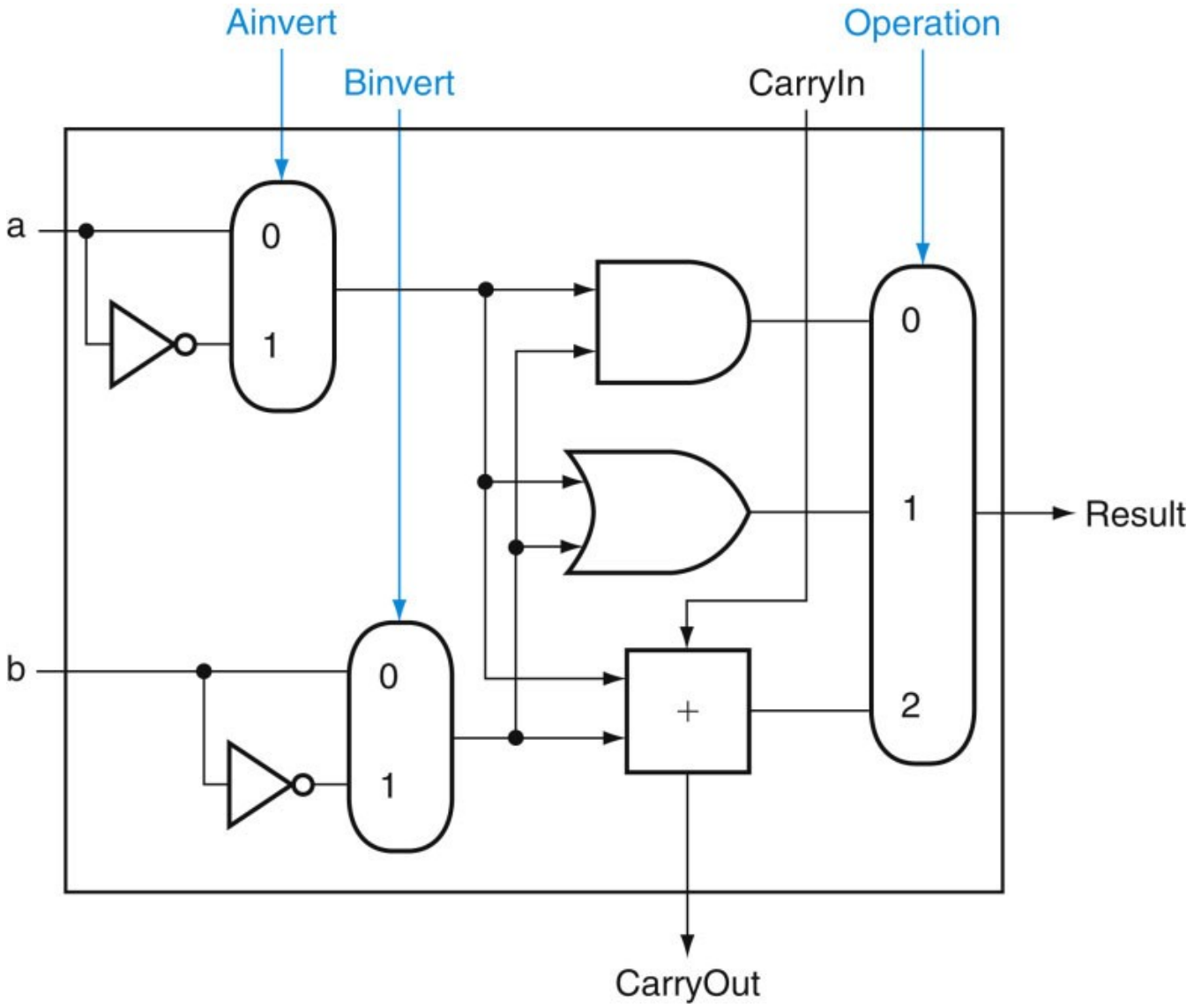
Incorporating NOR and NAND



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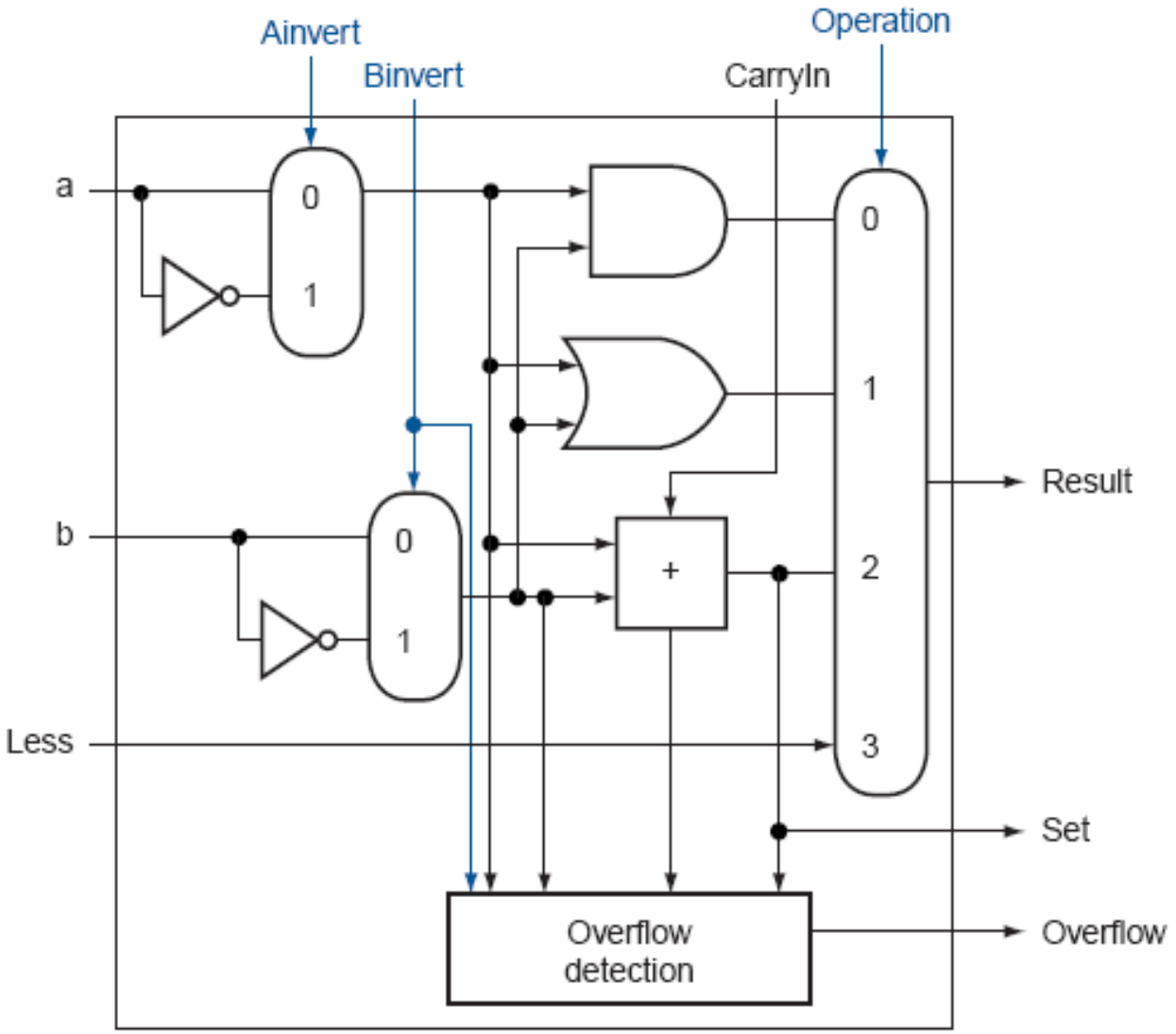
Incorporating cmp (set bit when less than)

- Perform a – b and check the sign
- New signal (Less) that is zero for ALU boxes 1-31
- The 31st box has a unit to detect overflow and sign – the sign bit serves as the Less signal for the 0th box



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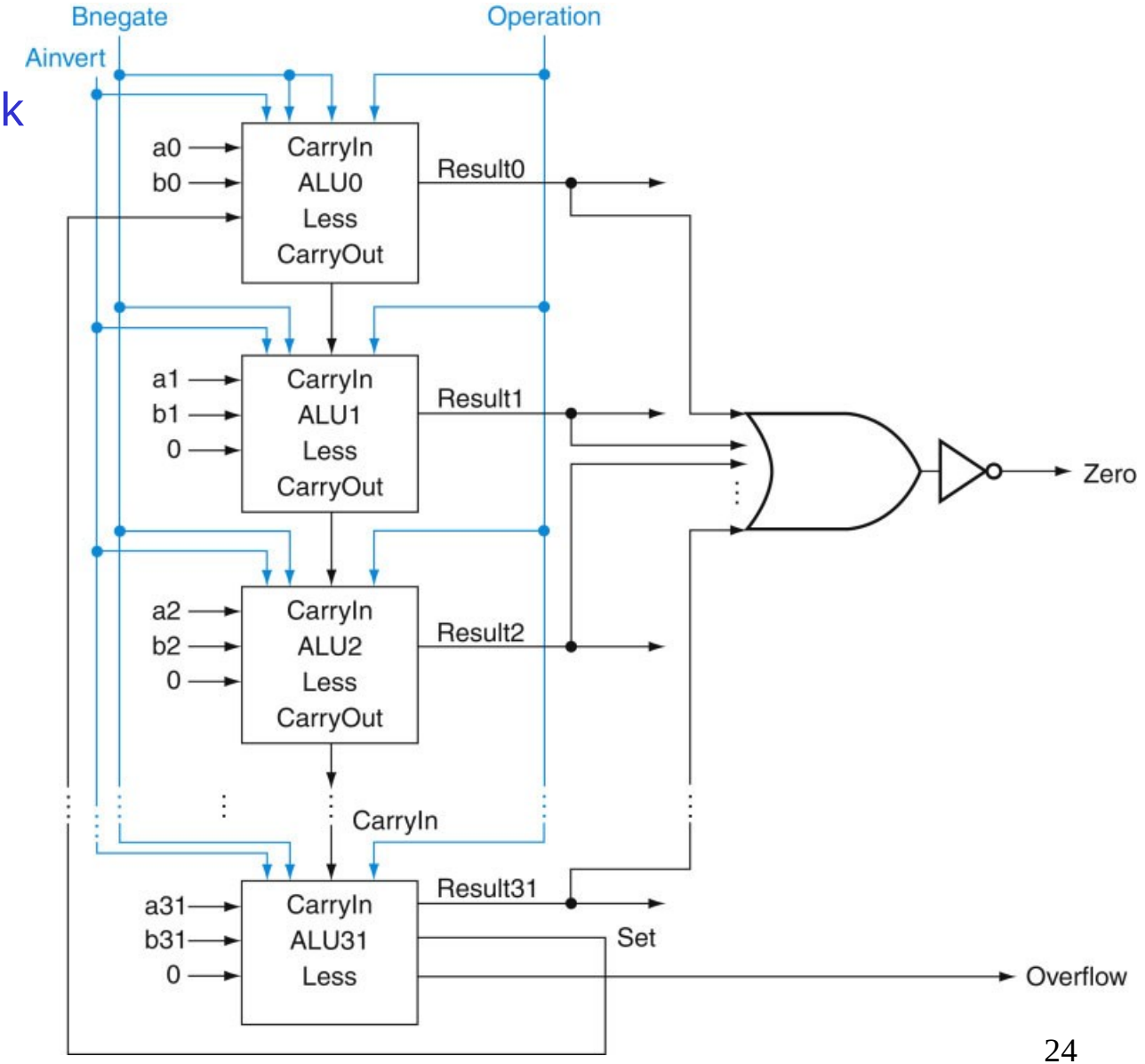

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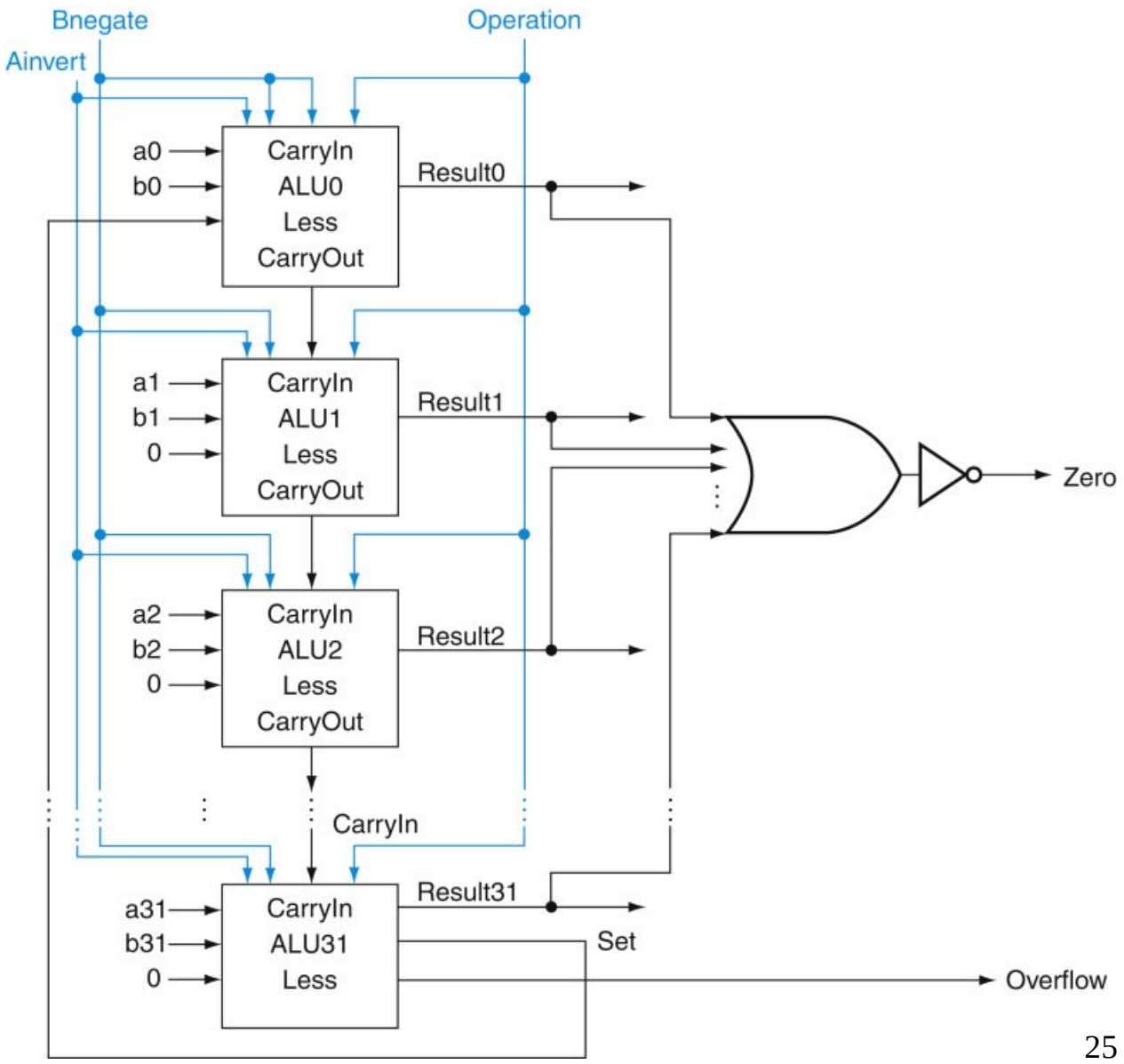
Incorporating jeq (jump when equal)

- Perform a – b and confirm that the result is all zero's



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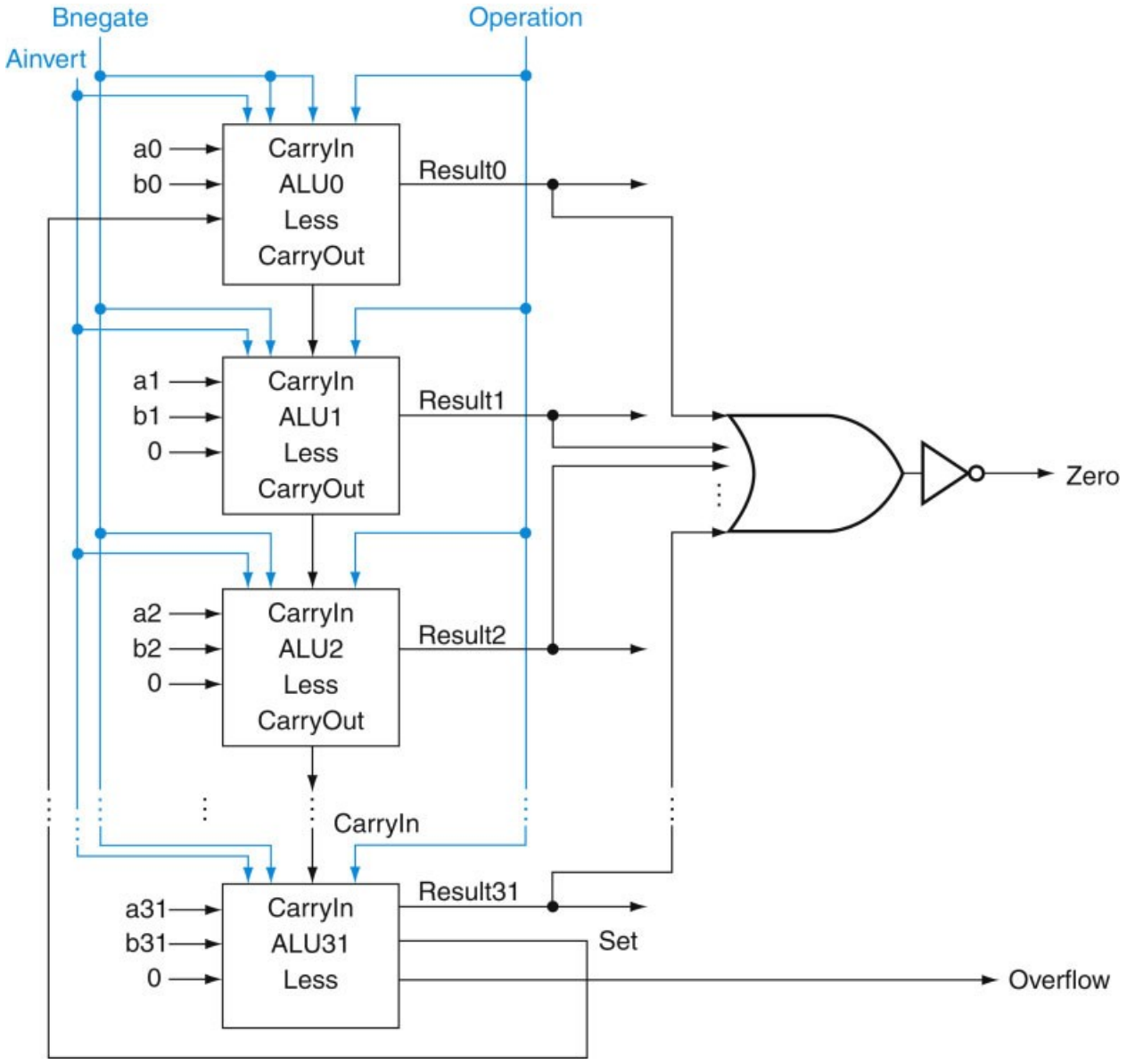
Control Lines

What are the values of the control lines and what operations do they correspond to?



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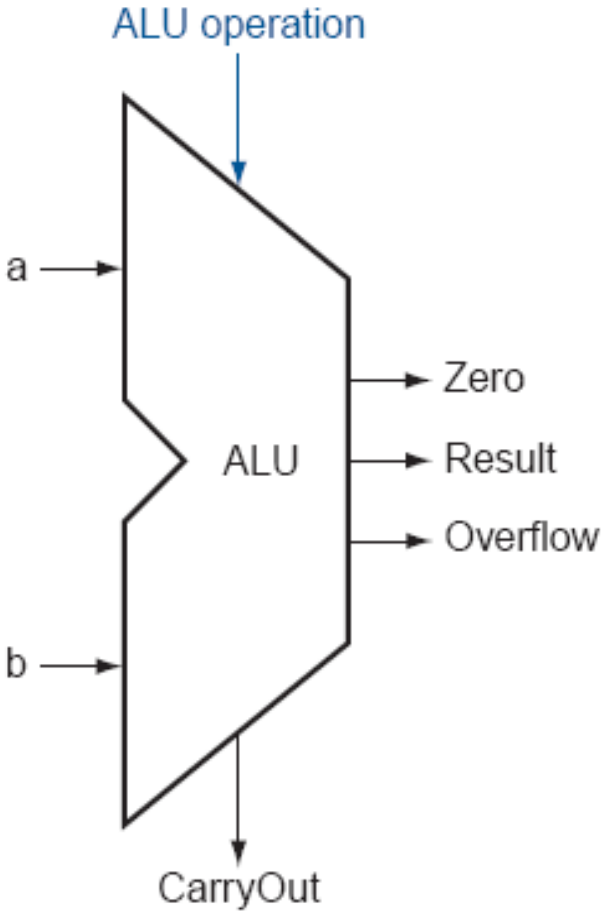
Control Lines

What are the values of the control lines and what operations do they correspond to?

	Ai	Bn	Op
AND	0	0	00
OR	0	0	01
Add	0	0	10
Sub	0	1	10
SLT	0	1	11
NOR	1	1	00



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Speed of Ripple Carry

- The carry propagates thru every 1-bit box: each 1-bit box sequentially implements AND and OR – total delay is the time to go through 64 gates!

- We've already seen that any logic equation can be expressed as the sum of products – so it should be possible to compute the result by going through only 2 gates!

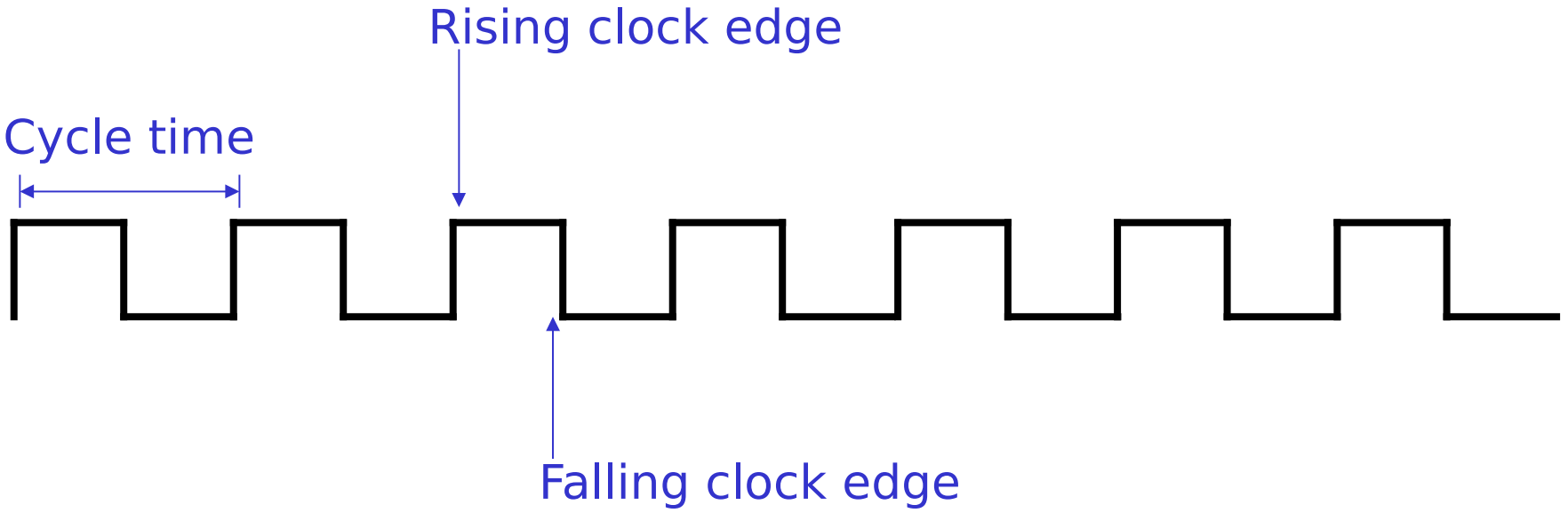
- Caveat: need many parallel gates and each gate may have a very large number of inputs – it is difficult to efficiently build such large gates, so we'll find a compromise:
  - moderate number of gates
  - moderate number of inputs to each gate
  - moderate number of sequential gates traversed

Clocks

- A microprocessor is composed of many different circuits that are operating simultaneously – if each circuit X takes ininputs at time TI_X, takes time TE_X to execute the logic, and produces outputs at time TO_X, imagine the complications in co-ordinating the tasks of every circuit

- A major school of thought (used in most processors built today): all circuits on the chip share a clock signal (a square wave) that tells every circuit when to accept inputs, how much time they have to execute the logic, and when they must produce outputs

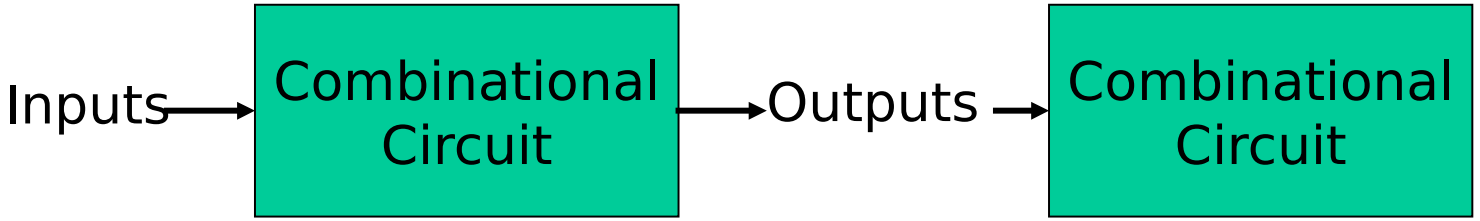
Clock Terminology



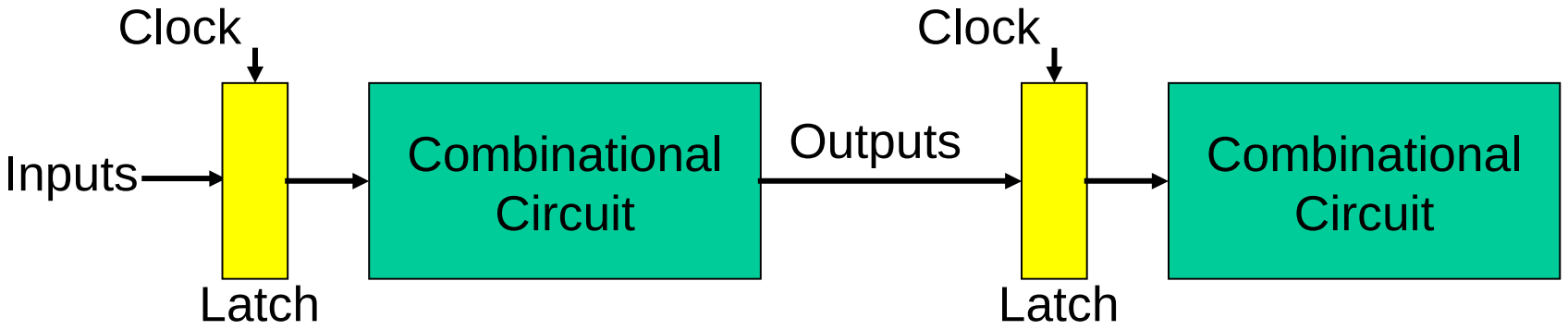
$$4\ \text{GHz} = \text{clock speed} = \frac{1}{\text{cycle time}} = \frac{1}{250\ \text{ps}}.$$

Sequential Circuits

- Until now, circuits were combinational – when inputs change, the outputs change after a while (time = logic delay thru circuit)



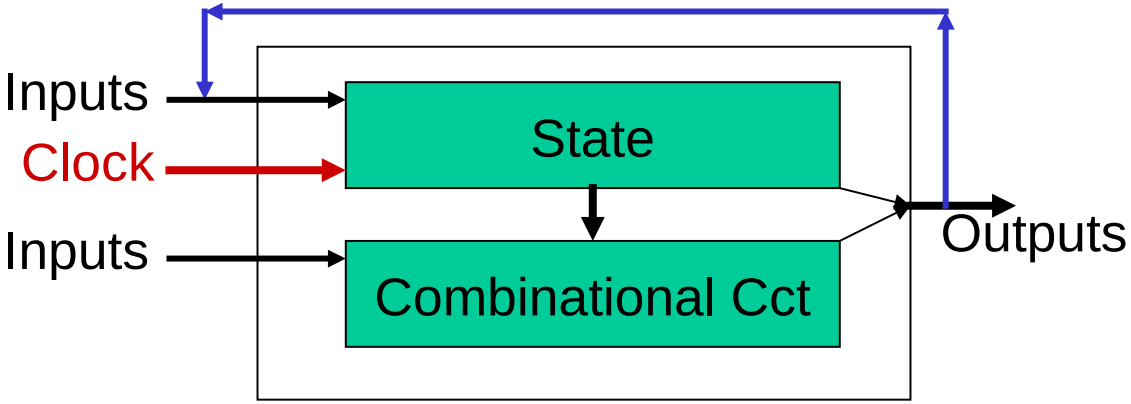
- We want the clock to act like a start and stop signal – a "latch" is a storage device that separates these circuits – it ensures that the inputs to the circuit do not change during a clock cycle



Thank you!

Sequential Circuits

- Sequential circuit: consists of combinational circuit and a storage element



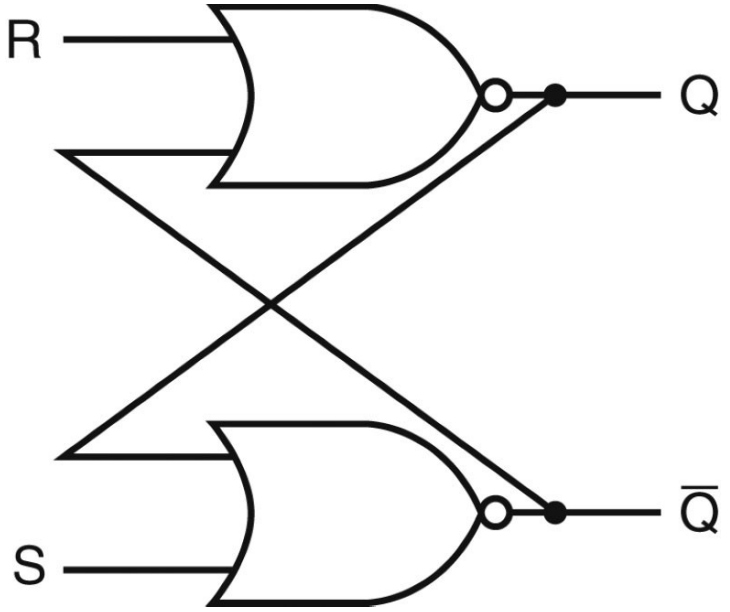
- At the start of the clock cycle, the rising edge causes the “state” storage to store some input values
- This state will not change for an entire cycle (until next rising edge)
- The combinational circuit has some time to accept the value of “state” and “inputs” and produce “outputs”
- Some of the outputs (for example, the value of next “state”) may feed back (but through the latch so they’re only seen in the next cycle)

Sequential circuits

Designing a Latch

- An S-R latch: set-reset latch
  - When Set is high, a 1 is stored
  - When Reset is high, a 0 is stored
  - When both are low, the previous state is preserved (hence, known as a storage or memory element)
  - Both are high – this set of inputs is not allowed

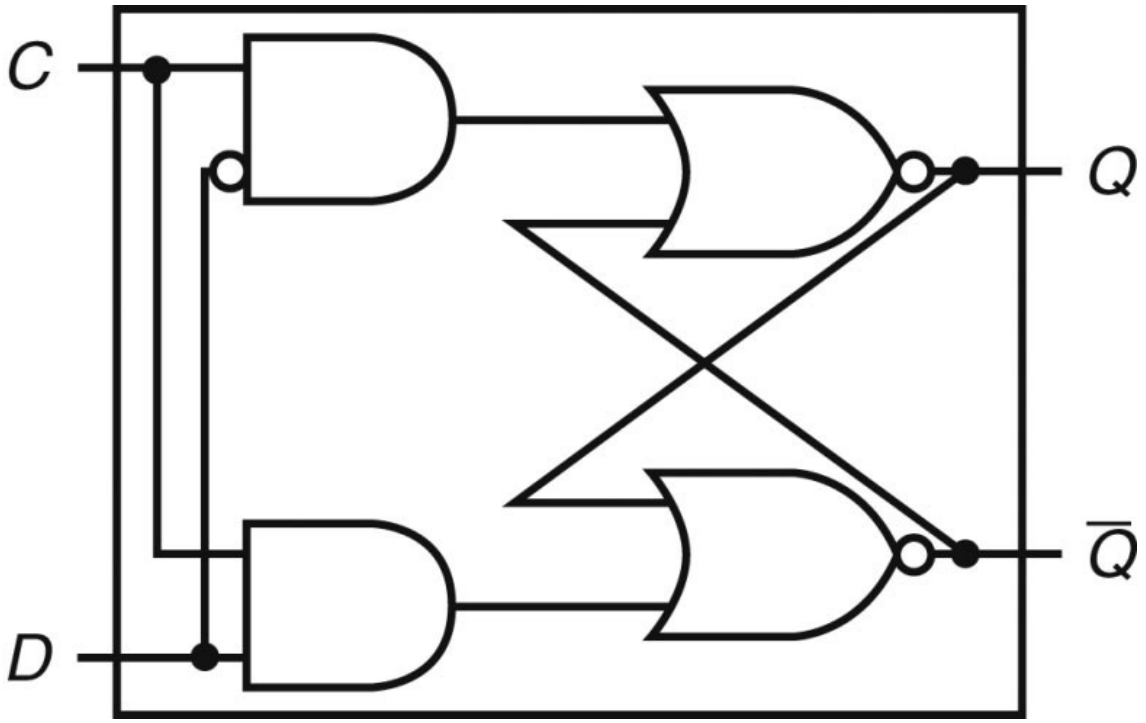
Verify the above behavior!



Source: H&P textbook

D Latch

- Incorporates a clock
- The value of the input D signal (data) is stored only when the clock is high – the previous state is preserved when the clock is low

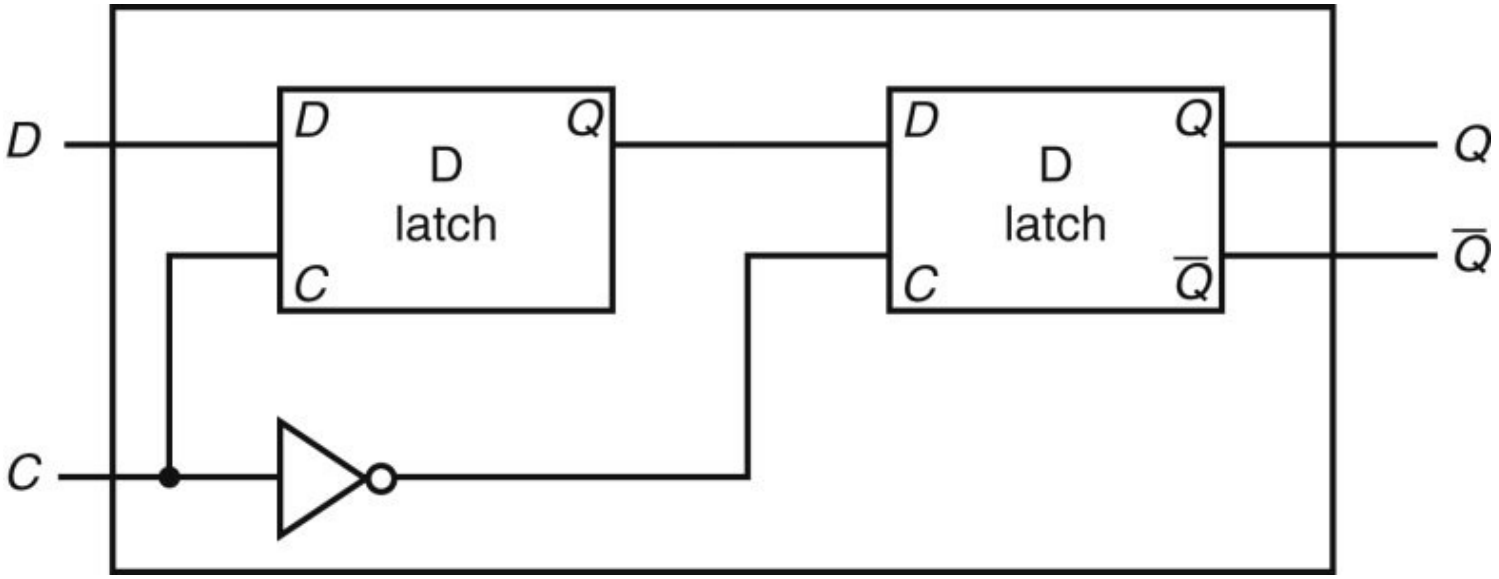


Source: H&P textbook

D Flip Flop

- Terminology:
  Latch: outputs can change any time the clock is high (asserted)
  Flip flop: outputs can change only on a clock edge

- Two D latches in series – ensures that a value is stored only on the falling edge of the clock



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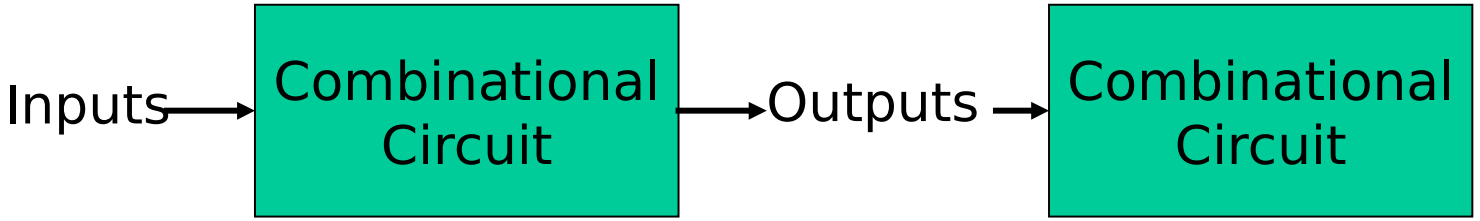

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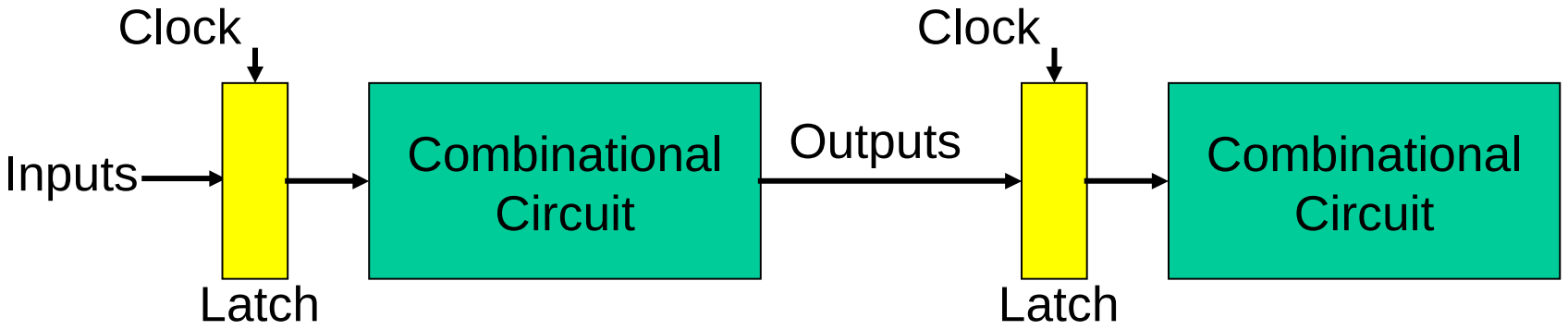
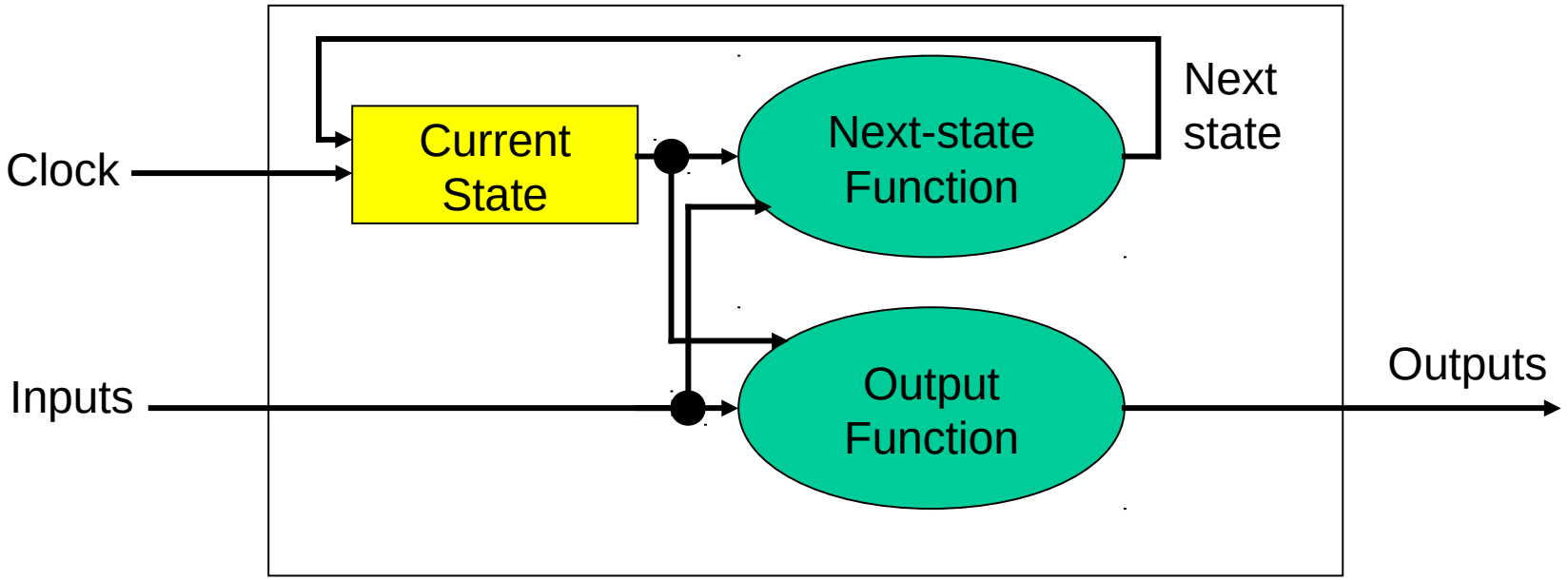


Finite State Machines

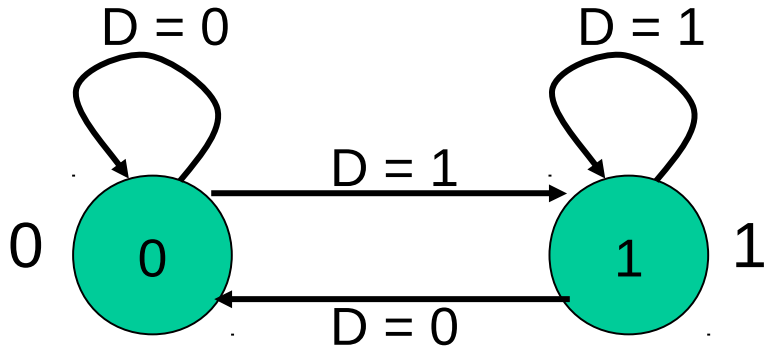
Finite State Machine

- A sequential circuit is described by a variation of a truth table – a finite state diagram (hence, the circuit is also called a finite state machine)
- Note that state is updated only on a clock edge



State Diagrams

- Each state is shown with a circle, labeled with the state value – the contents of the circle are the outputs
- An arc represents a transition to a different state, with the inputs indicated on the label



This is a state diagram for ___?

3-Bit Counter

- Consider a circuit that stores a number and increments the value on every clock edge – on reaching the largest value, it starts again from 0

  Draw the state diagram:
  - How many states?
  - How many inputs?

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Draw the state diagram:

- How many states?
- How many inputs?