How can Computational Geometry help Robotics and Automation:

Shorter, Smaller, Tighter – Old and New Challenges

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Algorithms in the Field/CG, CG Week
Chapel Hill, 2012

[UPenn, GRASP]

[Volkswagen]

Motion planning: the basic problem

Let B be a system (the robot) with k degrees of freedom moving in a known environment cluttered with obstacles. Given free start and goal placements for B decide whether there is a collision free motion for B from start to goal and if so plan such a motion.

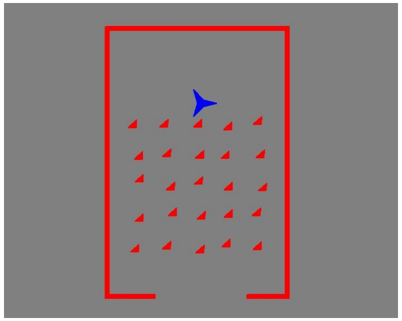
Two key terms: (i) degrees of freedom (dofs) and (ii) configuration space

The number of degrees of freedom (dofs)

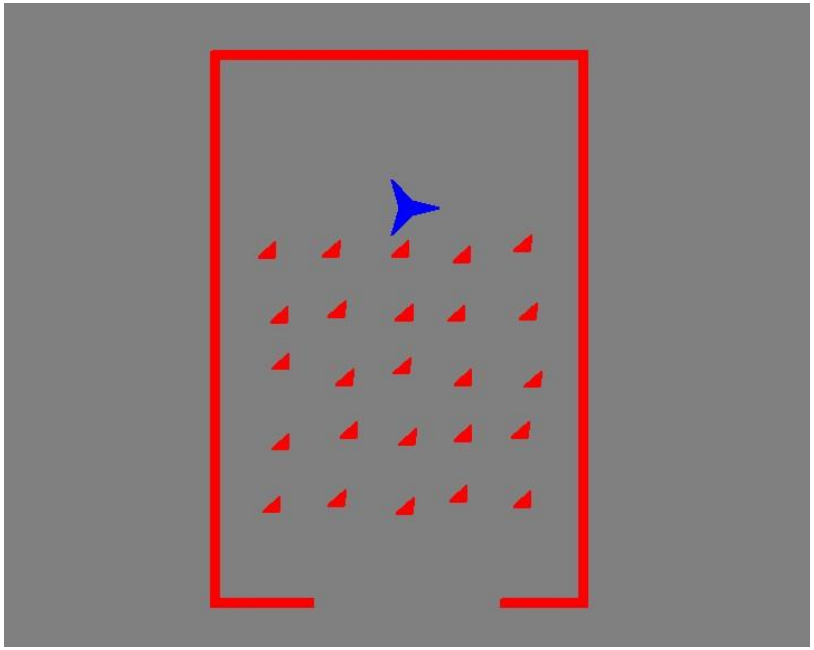
= *the number of independent parameters that define a configuration*

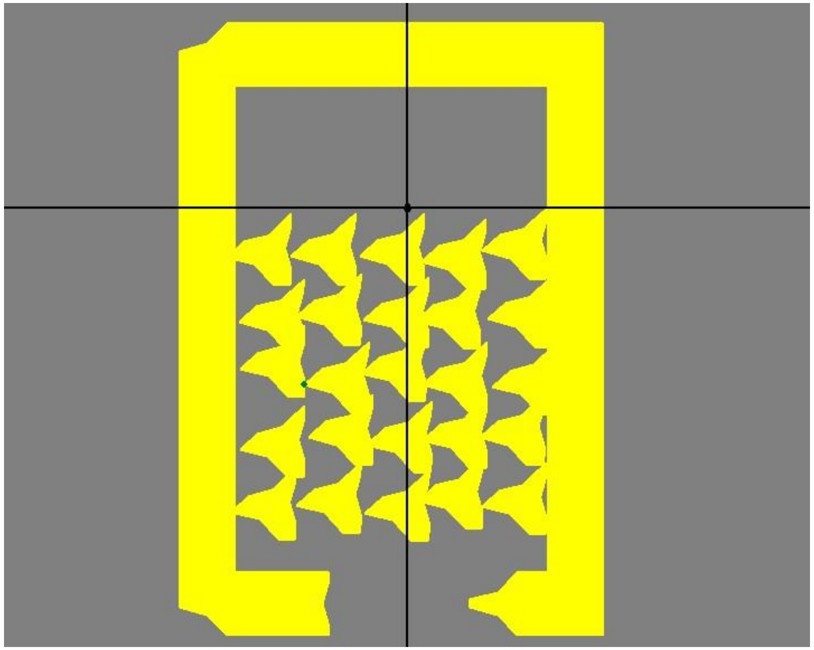
- a polygon robot translating in the plane 2
- a polygon robot translating and rotating 3
- a spatial robot translating and rotating 6
- industrial robot arms

typically 4 - 6



Configuration space





[Lozano-Perez, late 70s]

Talk overview

- CG and R&A, a very brief history
- Shorter
  - and other objectives: motion path optimization
- Smaller
  - new manufacturing processes at the micro level
  - the motion of molecules
  - swarms of robots
- Tighter
  - assembly planning
  - motion in tight quarters

CG and R&A:
terse history through the motion-planning lens

late 1970s: C-space, motion planning is hard

early 80s: piano movers, general solution 2-epx

mid 80s: roadmap/silhouette, general solution 1-exp, potential field

late 80s to mid 90s: near-optimal solutions for small # of dofs

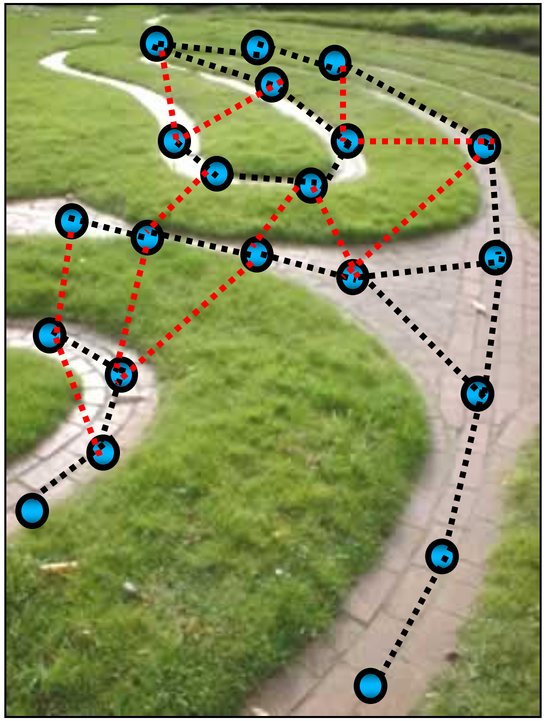
mid 90s: 1st WAFR (10th WAFR, last week)

mid 90s: PRM

…

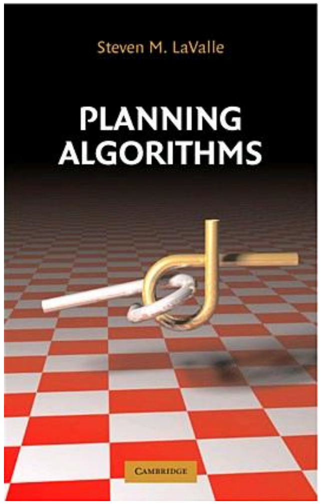
Sampling-based motion planners

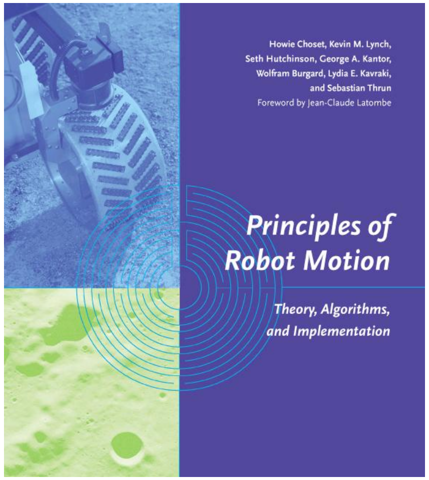
- PRM (Probabilistic RoadMaps) [Kavraki, Svestka, Latombe,Overmars 96]
- many variants followed, e.g. RRT (Rapidly Exploring Random trees), [LaValle-Kuffner 99,00]



Sampling-based motion planners, advantages

- easy to implement, provided you have a good static collision detector [Lin,Manocha et al; survey, Hdbk of DCG `04]
- extended the applicability of motion planning: animation, docking motions, virtual prototyping, more
- revealed the nature of many practical problems: dofs vs. tightness





side note

a (hidden?) gem:

Helmut Alt, Rudolf Fleischer, Michael Kaufmann, Kurt Mehlhorn, Stefan Näher, Stefan Schirra, Christian Uhrig: Approximate Motion Planning and the Complexity of the Boundary of the Union of Simple Geometric Figures Algorithmica 8(5&6): 391-406 (1992)

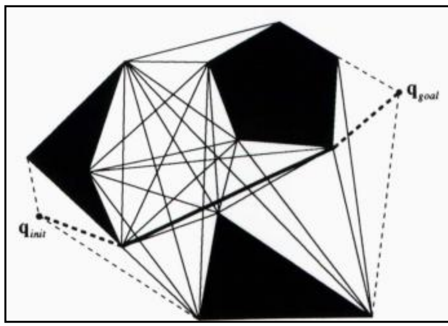
Sampling-based motion planners, shortcomings

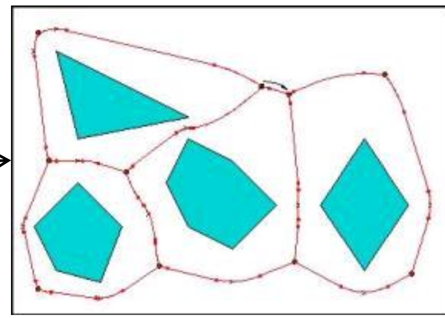
- path quality
- predictability or (in)operability in tight settings, the narrow passage problem

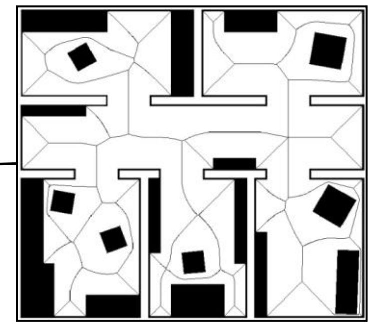
Shorter

motion path optimization

High-quality paths:
analytic solutions for simple cases







http://www.sfbtr8.uni-bremen.de/project/r3/HGVG/hierarchicalVGraphs.html

shortest path in 2D: Visibility Graph *(Nilsson '69, Lee '78, Hershberger and Suri '97)*

short + high clearance in 2D: Visibility-Voronoi Complex *(Wein et al., '07)*

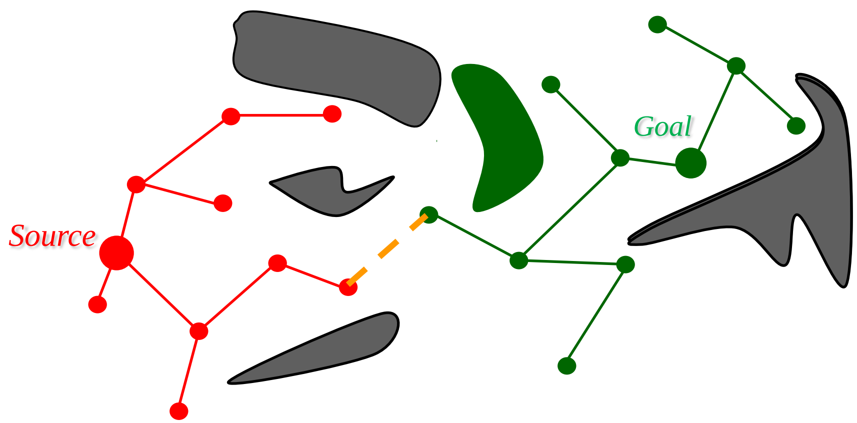
maximal clearance in 2D: Voronoi diagram *(O'dunlang and Yap, '82)*

but NP-hard in other settings with only a few degrees of freedom *(e.g., Canny and Reif, '87)*

Growing two-trees (Bi-RRT)

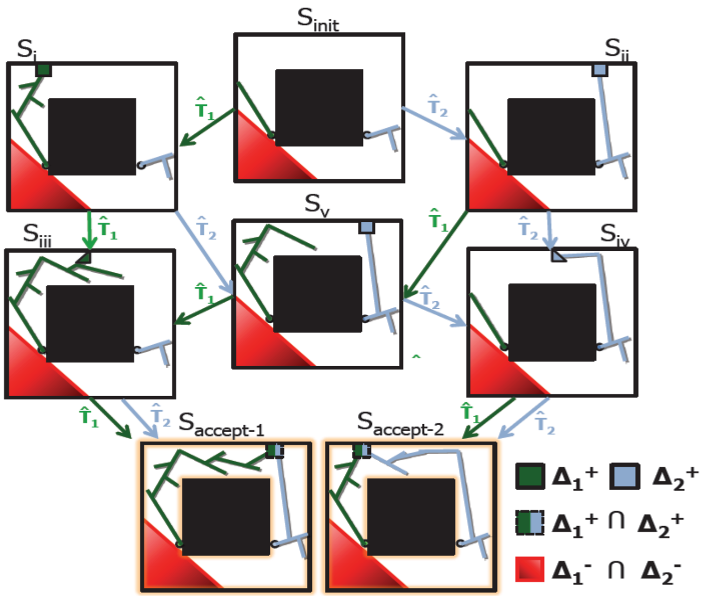
[Kuffner and LaValle ’00]

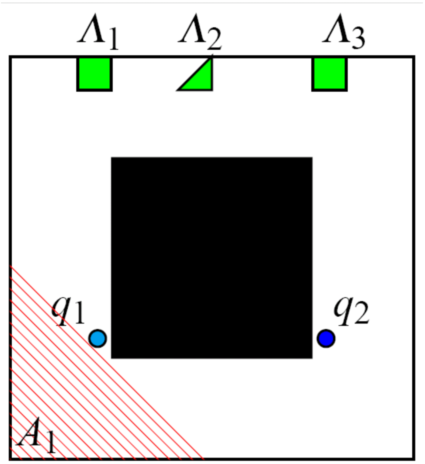
- maintain two trees rooted at source & goal
- construction step –
  sample configurations and expand either tree as in RRT
- merging step –
  connect configurations from both trees



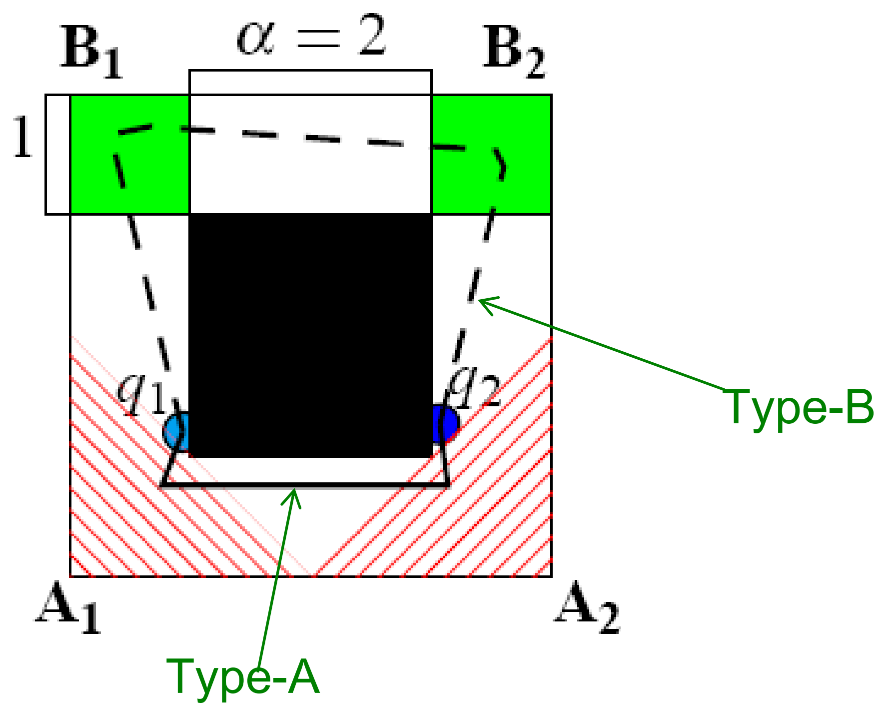
How low can path quality get?

Sampling-Diagram Automata:
Analysis of path quality in tree planners
[Nechushtan-Raveh-Halperin, WAFR 2010]



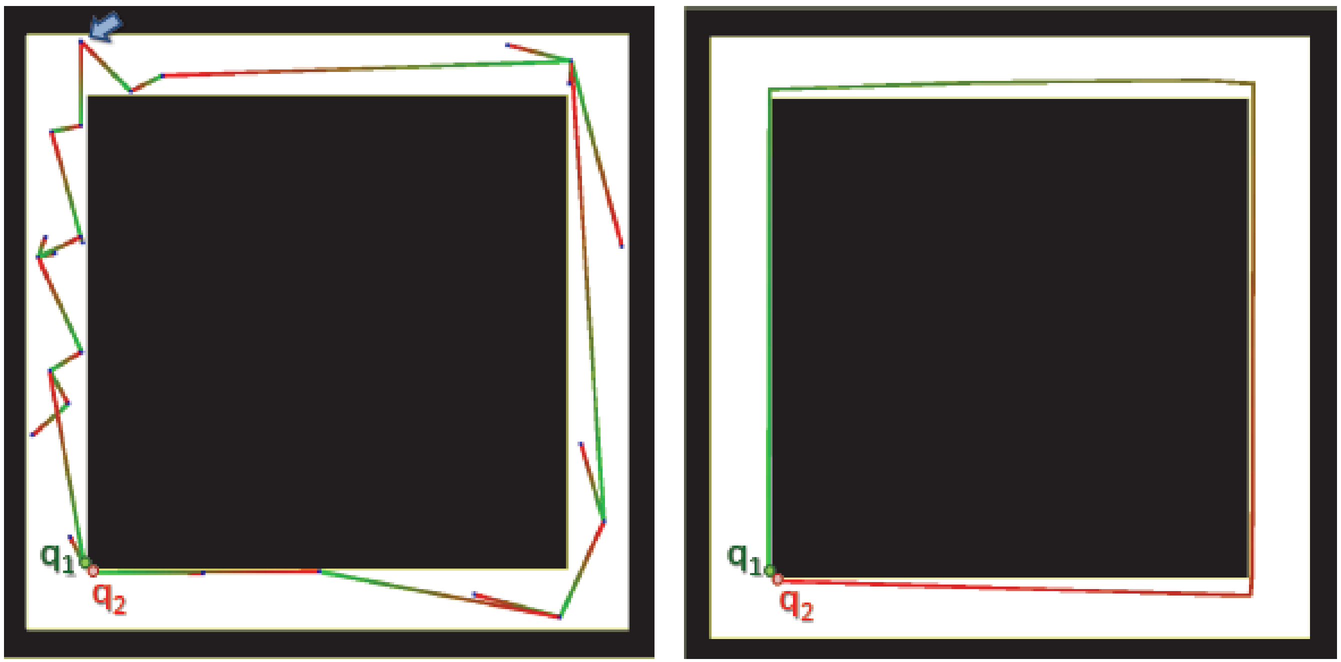


Experiments (I) – in OOPSMP



- 49.4% of paths are <u>over three times</u> worse than optimal (even after smoothing)
- much larger than the theoretical bound

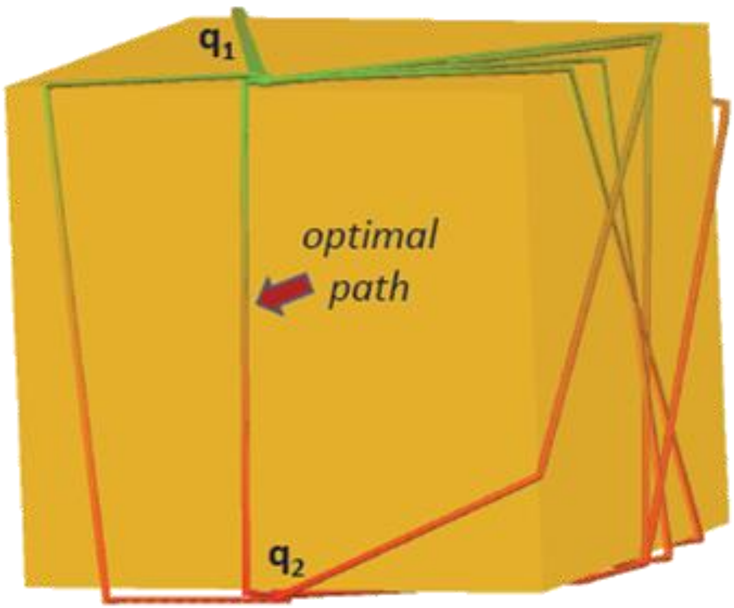
Experiments (II) – close-by start and goal configurations



- ❑ 5.9% of paths are <u>over 140 times</u> worse than optimal (even after smoothing)
- ❑ importance of *visibility blocking* – narrow passages not the only king

  (theoretical motivation for Visibility PRM, Laumond *et al.* ‘00)

Experiments (III) – 3D



Cube-within-Cube

Experiments: 97.3% (!) of paths are much worse than optimal <u>after smoothing</u>

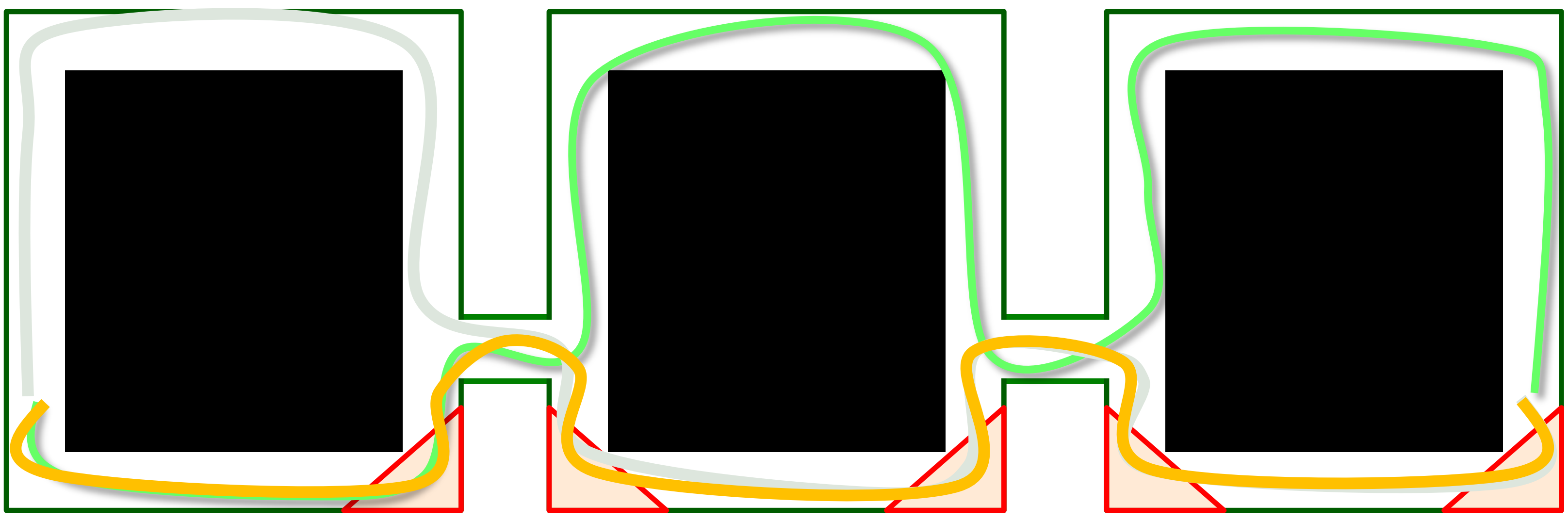
Improving path quality in sampling-based motion planning, related work

- Short-cutting heuristics (“path smoothing”)
- Retraction towards medial axis
  [e.g., Wilmarth et al. ‘99, Geraerts and Overmars ’07]
- Useful Cycles in PRM [Nieuwenhuisen and Overmars ’04]
- Biasing tree growth by a cost-function
  [e.g., Urmson and Simmons ‘03, Ettlin and Bleuler ‘06,
  Jaillet et al. ‘08, Raveh et al. ’09]
- Anytime RRT [Ferguson and Stentz ’06]

- RRT* - a modification of RRT [Karaman and Frazzoli ’10]
  - ➔ the modified **RRT*** algorithm converges to an optimal path as running time reaches infinity
  - ➔ “Standard”-RRT <u>misses</u> the (precise) optimal path with probability one
    Still, might be ε-good, or within same homotopy class as optimal path

More complex settings

Several visibility-blocking regions + repetitive structure

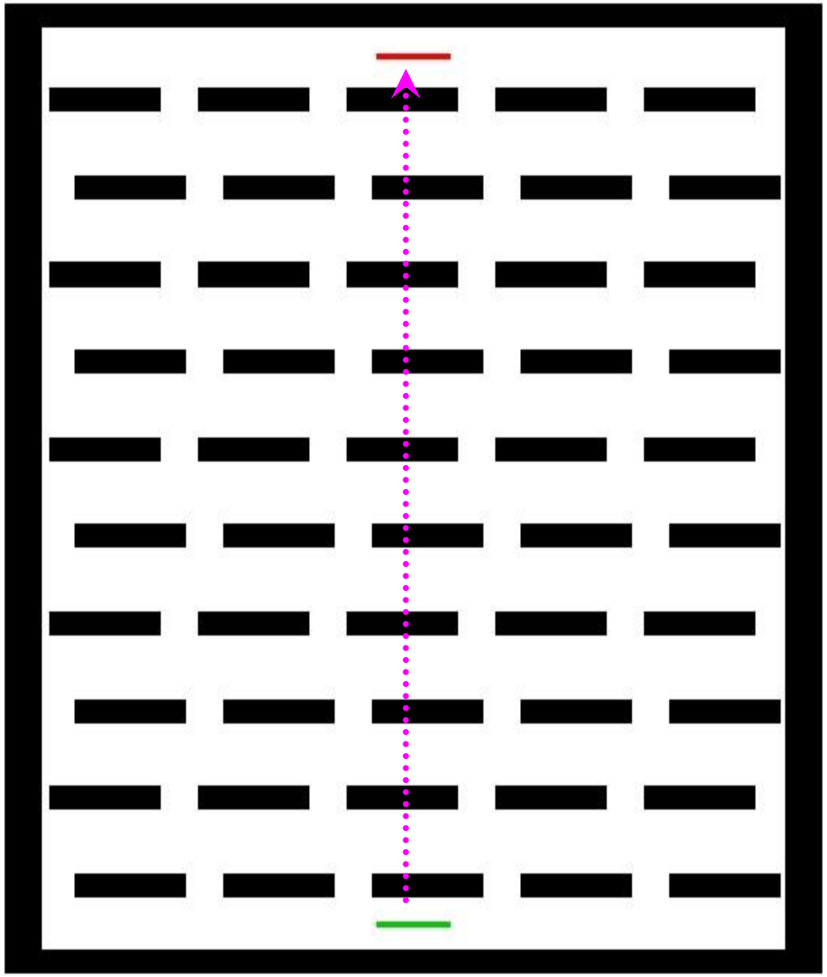


➔ wrong decision can be taken at every step
➔ can be solved by path-hybridization

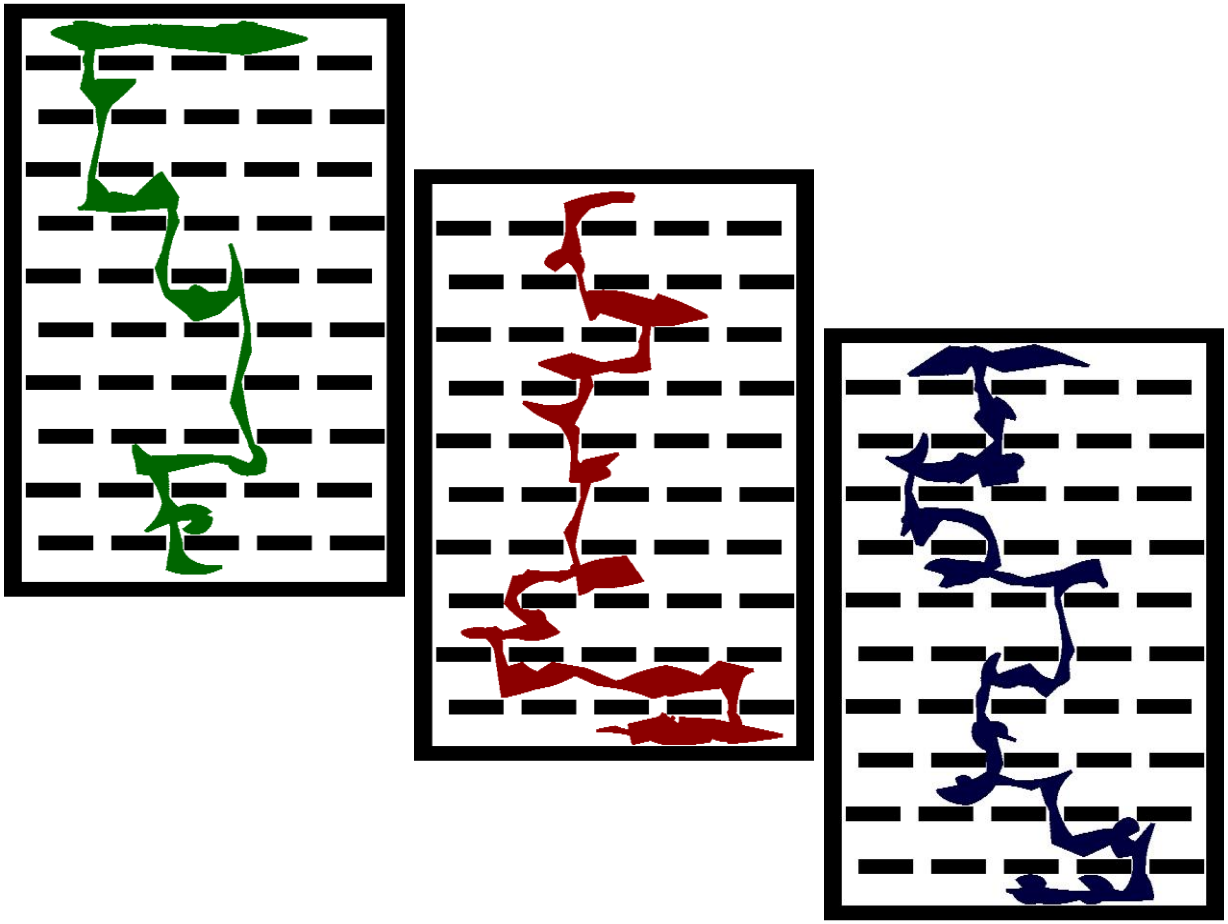
Improving quality by path hybridization

[Raveh,Enosh,H ‘11]

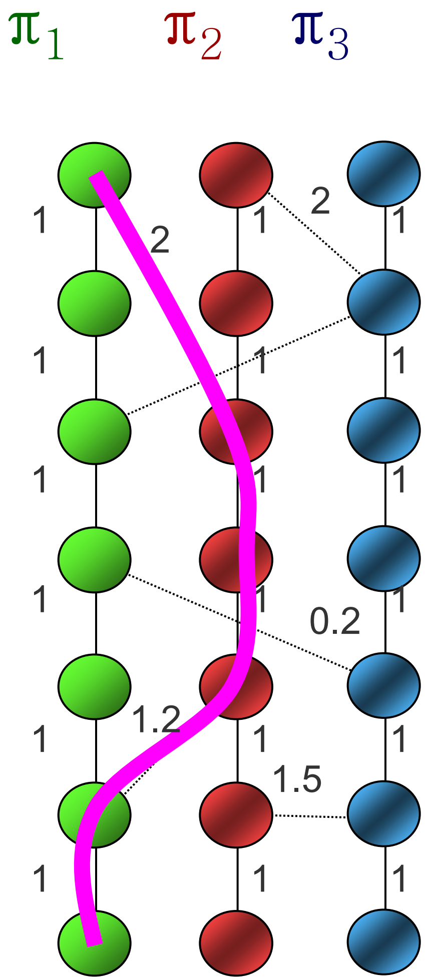
example: move the rod from the bottom to the top of a 2D grid *(rotation + translation)*



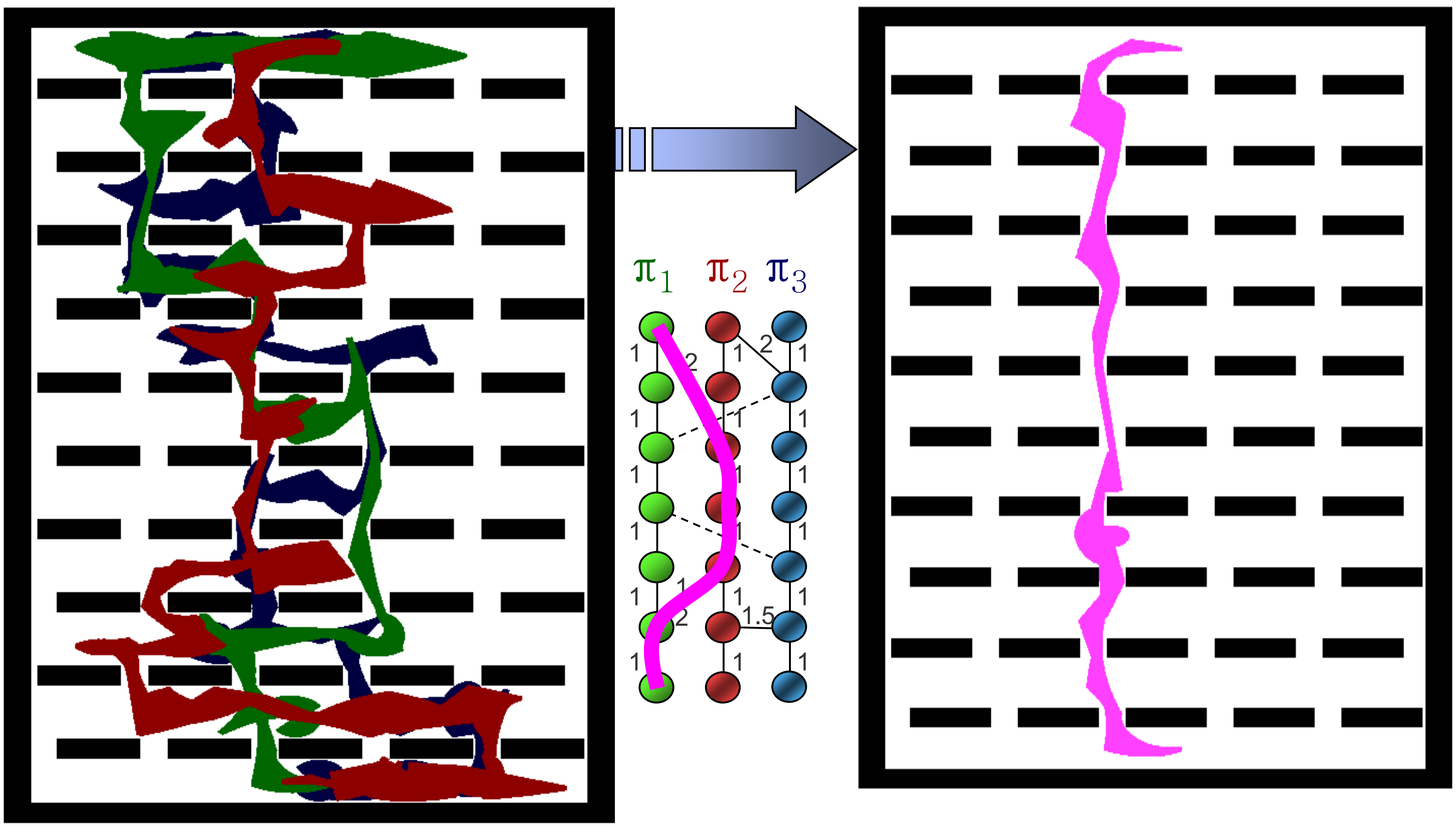
3 randomly generated motion paths

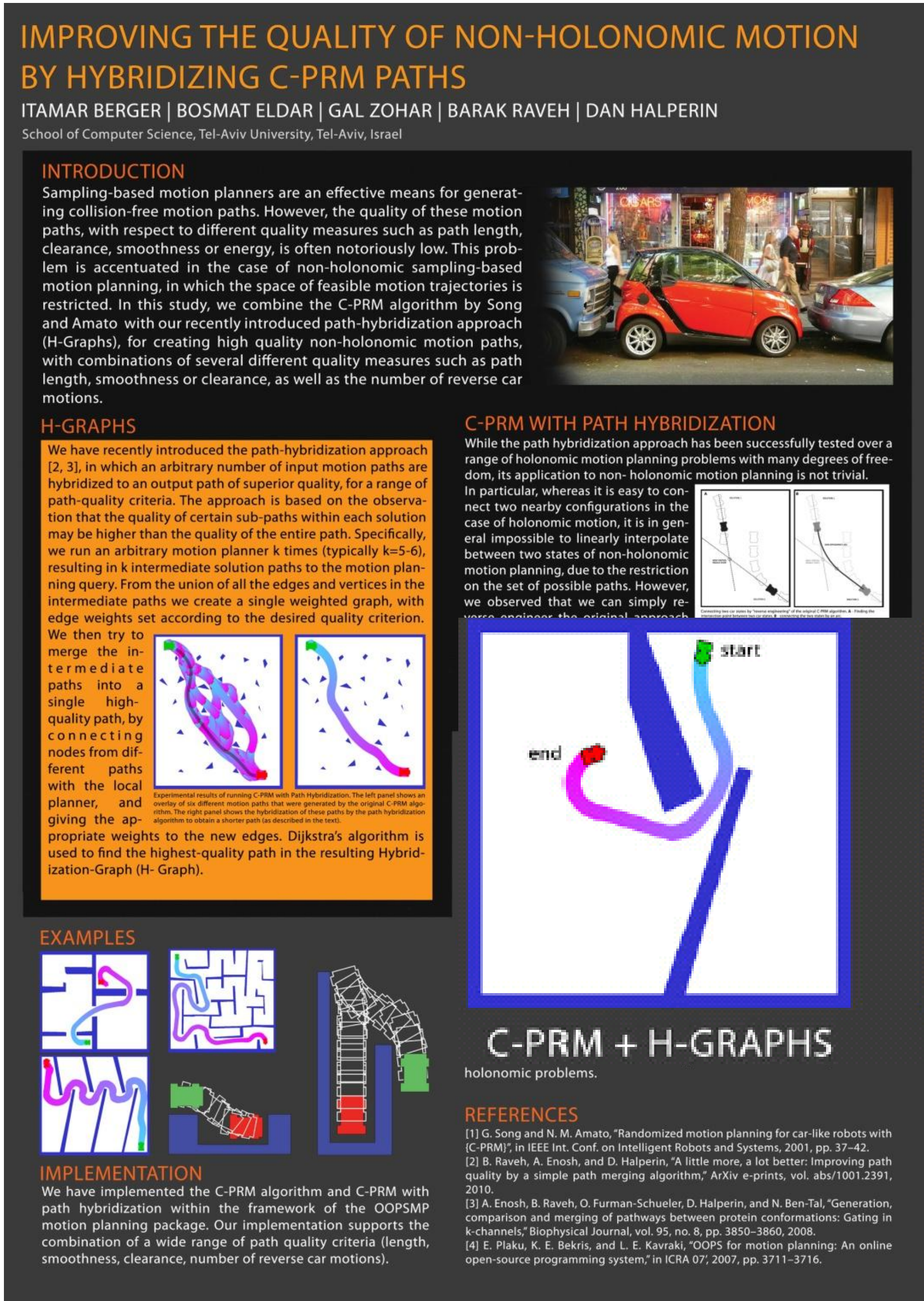


H-Graphs: Hybridizing multiple motion paths
(= looking for shortcuts)



Hybridizing the paths





IMPROVING THE QUALITY OF NON-HOLONOMIC MOTION BY HYBRIDIZING C-PRM PATHS

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INTRODUCTION

Sampling-based motion planners are an effective means for generating collision-free motion paths. However, the quality of these motion paths, with respect to different quality measures such as path length, clearance, smoothness or energy, is often notoriously low. This problem is accentuated in the case of non-holonomic sampling-based motion planning, in which the space of feasible motion trajectories is restricted. In this study, we combine the C-PRM algorithm by Song and Amato with our recently introduced path-hybridization approach (H-Graphs), for creating high quality non-holonomic motion paths, with combinations of several different quality measures such as path length, smoothness or clearance, as well as the number of reverse car motions.

H-GRAPHS

We have recently introduced the path-hybridization approach [2, 3], in which an arbitrary number of input motion paths are hybridized to an output path of superior quality, for a range of path-quality criteria. The approach is based on the observation that the quality of certain sub-paths within each solution may be higher than the quality of the entire path. Specifically, we run an arbitrary motion planner k times (typically k=5-6), resulting in k intermediate solution paths to the motion planning query. From the union of all the edges and vertices in the intermediate paths we create a single weighted graph, with edge weights set according to the desired quality criterion. We then try to merge the intermediate paths into a single high-quality path, by connecting nodes from different paths with the local planner, and giving the appropriate weights to the new edges. Dijkstra's algorithm is used to find the highest-quality path in the resulting Hybridization-Graph (H- Graph).

Experimental results of running C-PRM with Path Hybridization. The left panel shows an overlay of six different motion paths that were generated by the original C-PRM algorithm. The right panel shows the hybridization of these paths by the path hybridization algorithm to obtain a shorter path (as described in the text).

C-PRM WITH PATH HYBRIDIZATION

While the path hybridization approach has been successfully tested over a range of holonomic motion planning problems with many degrees of freedom, its application to non- holonomic motion planning is not trivial. In particular, whereas it is easy to connect two nearby configurations in the case of holonomic motion, it is in general impossible to linearly interpolate between two states of non-holonomic motion planning, due to the restriction on the set of possible paths. However, we observed that we can simply re-

holonomic problems.

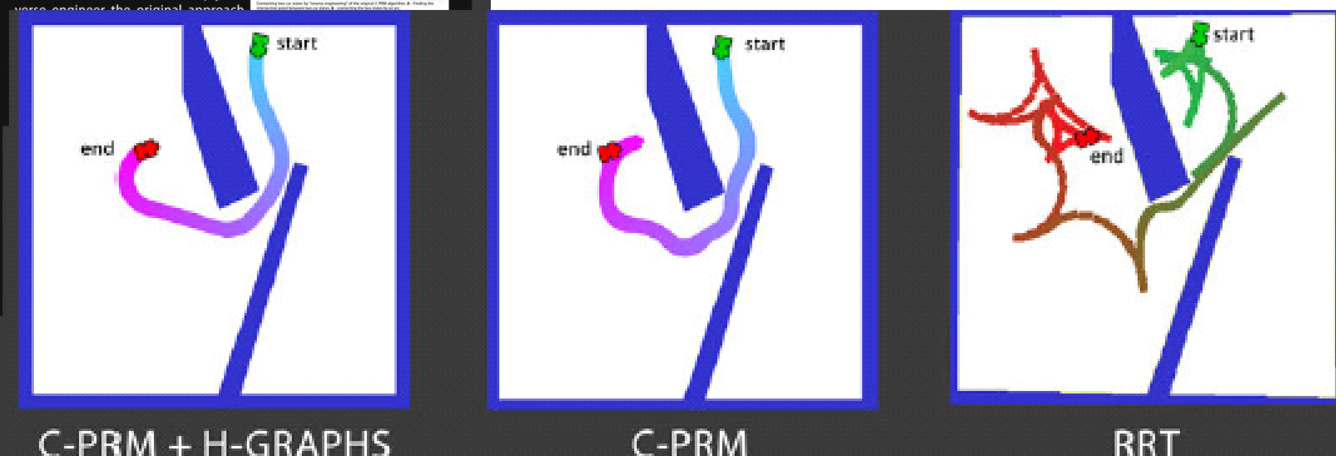
EXAMPLES

IMPLEMENTATION

We have implemented the C-PRM algorithm and C-PRM with path hybridization within the framework of the OOPSMP motion planning package. Our implementation supports the combination of a wide range of path quality criteria (length, smoothness, clearance, number of reverse car motions).

REFERENCES

[1] G. Song and N. M. Amato, "Randomized motion planning for car-like robots with (C-PRM)", in IEEE Int. Conf. on Intelligent Robots and Systems, 2001, pp. 37–42.
[2] B. Raveh, A. Enosh, and D. Halperin, "A little more, a lot better: Improving path quality by a simple path merging algorithm," ArXiv e-prints, vol. abs/1001.2391, 2010.
[3] A. Enosh, B. Raveh, O. Furman-Schueler, D. Halperin, and N. Ben-Tal, "Generation, comparison and merging of pathways between protein conformations: Gating in k-channels," Biophysical Journal, vol. 95, no. 8, pp. 3850–3860, 2008.
[4] E. Plaku, K. E. Bekris, and L. E. Kavraki, "OOPS for motion planning: An online open-source programming system," in ICRA 07', 2007, pp. 3711–3716.

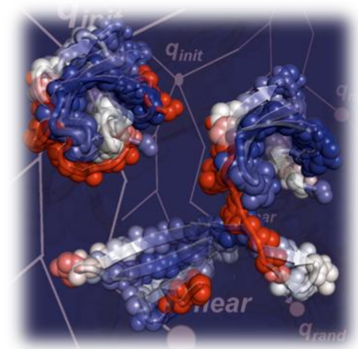


applied to car-like motion with various quality criteria: length, smoothness, clearance, number of reverse vehicle motions

Path quality, a few challenges

- further analysis of various quality criteria in sampling-based planners [Frazzoli-Karaman, Nechushtan et al]
- certified approximation [Clarkson, Agarwal et al], multi-objective structures for more dofs [visibility-Voronoi, Wein et al]
- roadmap size reduction while keeping path quality, spanner-style [Bekris et al]
- system-tailored optimization



Smaller

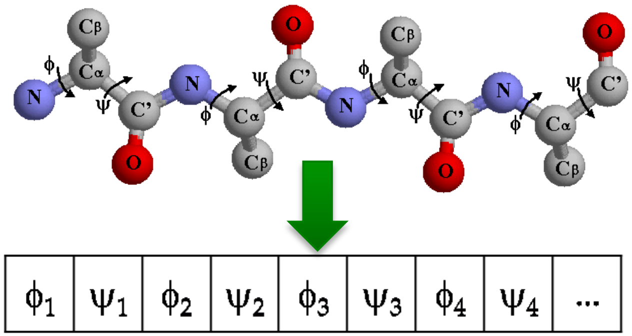
three little pieces
on big problems

Small I: nanomanufacturing and an inverse Voronoi challenge

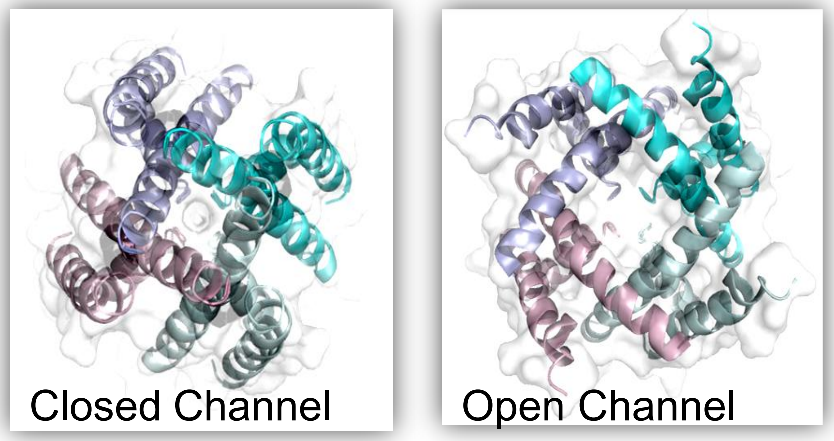
Slides by Karl Boehringer

http://acg.cs.tau.ac.il/courses/computational-geometry/spring-2012/guest_lecture_karl_nanomanufactoring.pdf

Small II: simulation and prediction of molecular motion (proteins as tiny robots)



[Lotan et al]

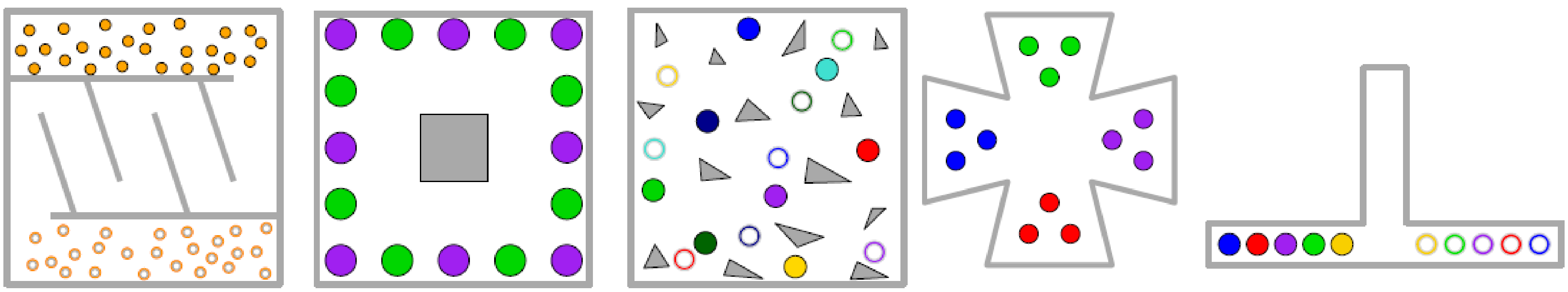


[Raveh et al]

challenges:

- handling thousands of dofs
- fast dynamic data structures for collision detection and energy recalculation under conformational changes

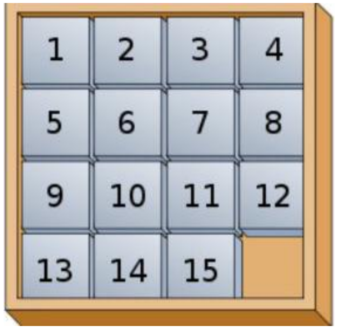
Small III (and big): multi-robot coordination



[Solovey-H]

challenges:

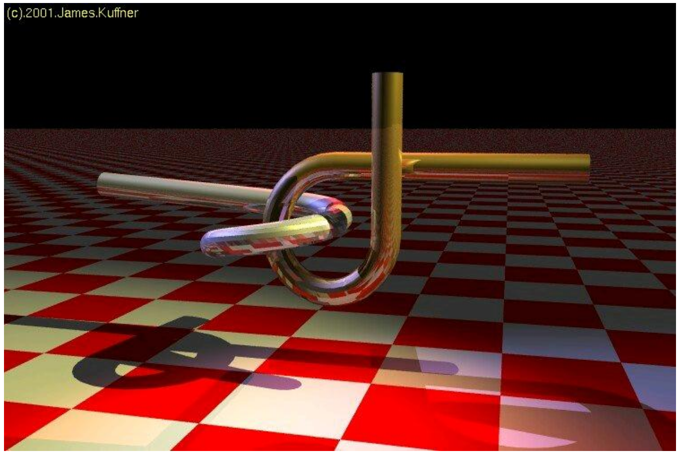
- effective planners with guarantees
- the k-color variant
- optimization



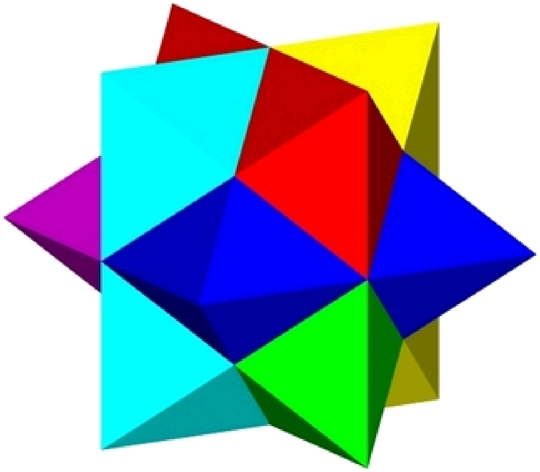
Tighter

motion and production
in tight settings

Movable separability* and assembly planning



[www.kuffner.org]

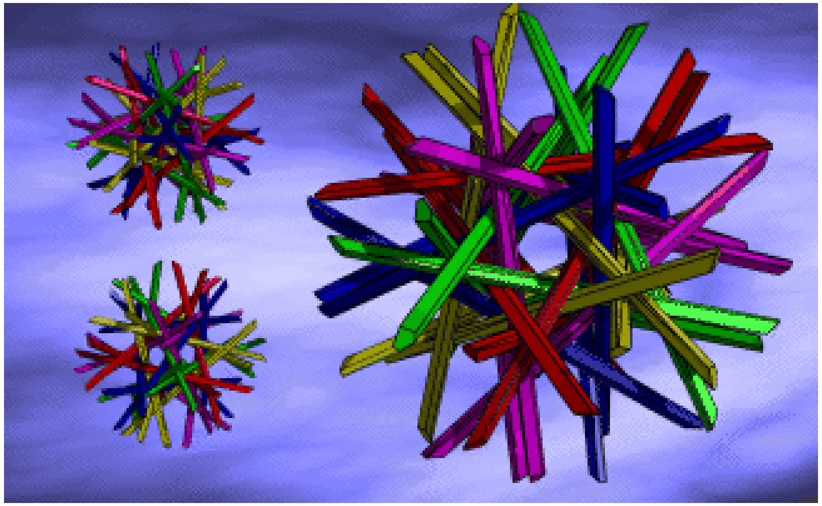


[Fogel-H]

* G. Toussaint

Convex objects

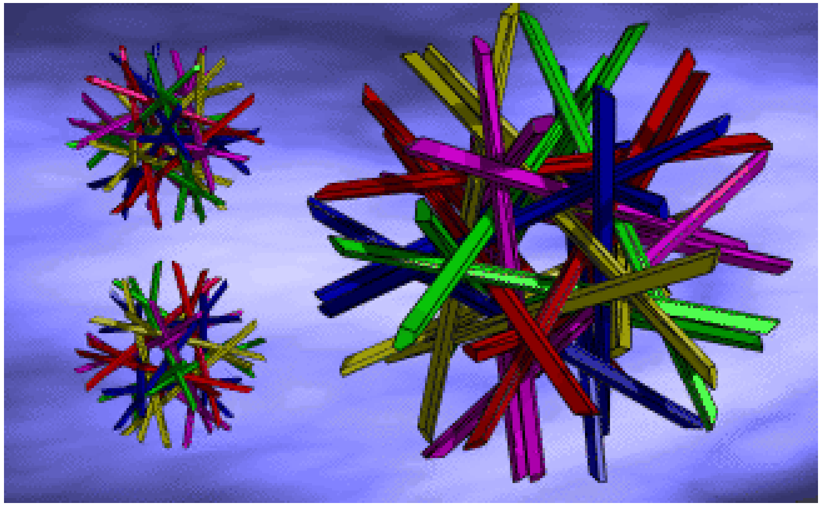
- in the plane: admit a disassembly sequence translating one part at a time along a fixed (arbitrary) direction to infinity [Guibas-Yao '80]
- in 3-space?
  - depth order does not always exist
  - moreover, assemblies of convex parts may be interlocked



[Snoeyink-Stolfi 93]

side note

The assembly in the figure [Snoeyink-Stolfi 93] cannot be taken apart with two hands and consists of thirty (30) convex parts. Is there an assembly with fewer convex parts that cannot be taken apart?



The partitioning problem, hardness

- arbitrary motions: assembly partitioning for polyhedral parts of constant maximum complexity each is PSPACE-hard

- 2-handed assembly partitioning for polygonal parts with translational motions only and into connected subassemblies is NP-complete [Kavraki-Kolountzakis ’95]

General framework for assembly planning

- The non-directional blocking graph [Wilson-Latombe `94]:

For fixed complexity motion types assembly planning is polynomial!

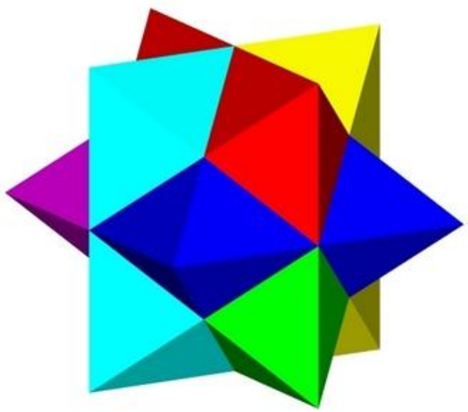
Archimedes
[Wilson et al`97]

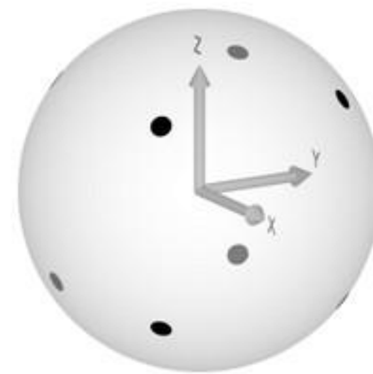
- The motion space approach [H-Latombe-Wilson `98]

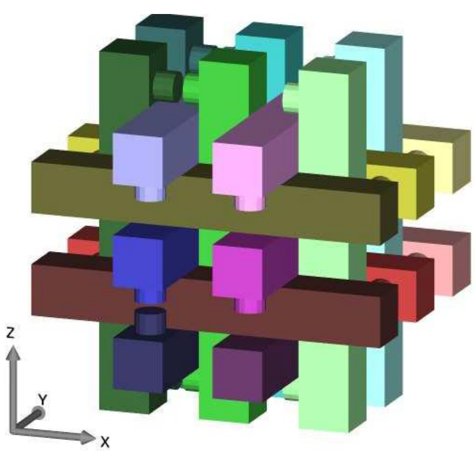
The critical factor is the dimension of the motion space, so far

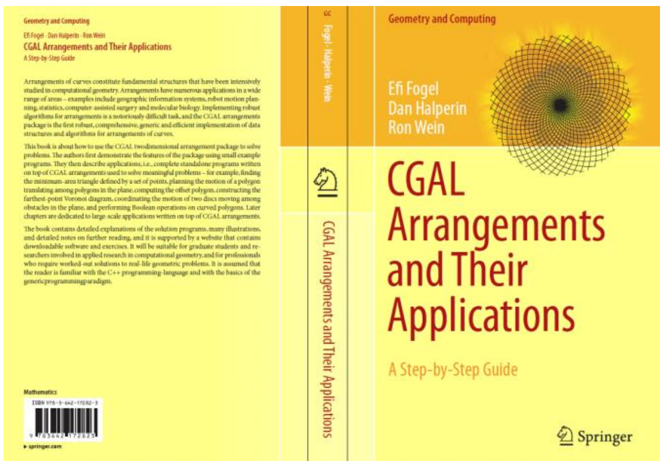
Assembly partitioning with infinite translations

[Fogel-H]





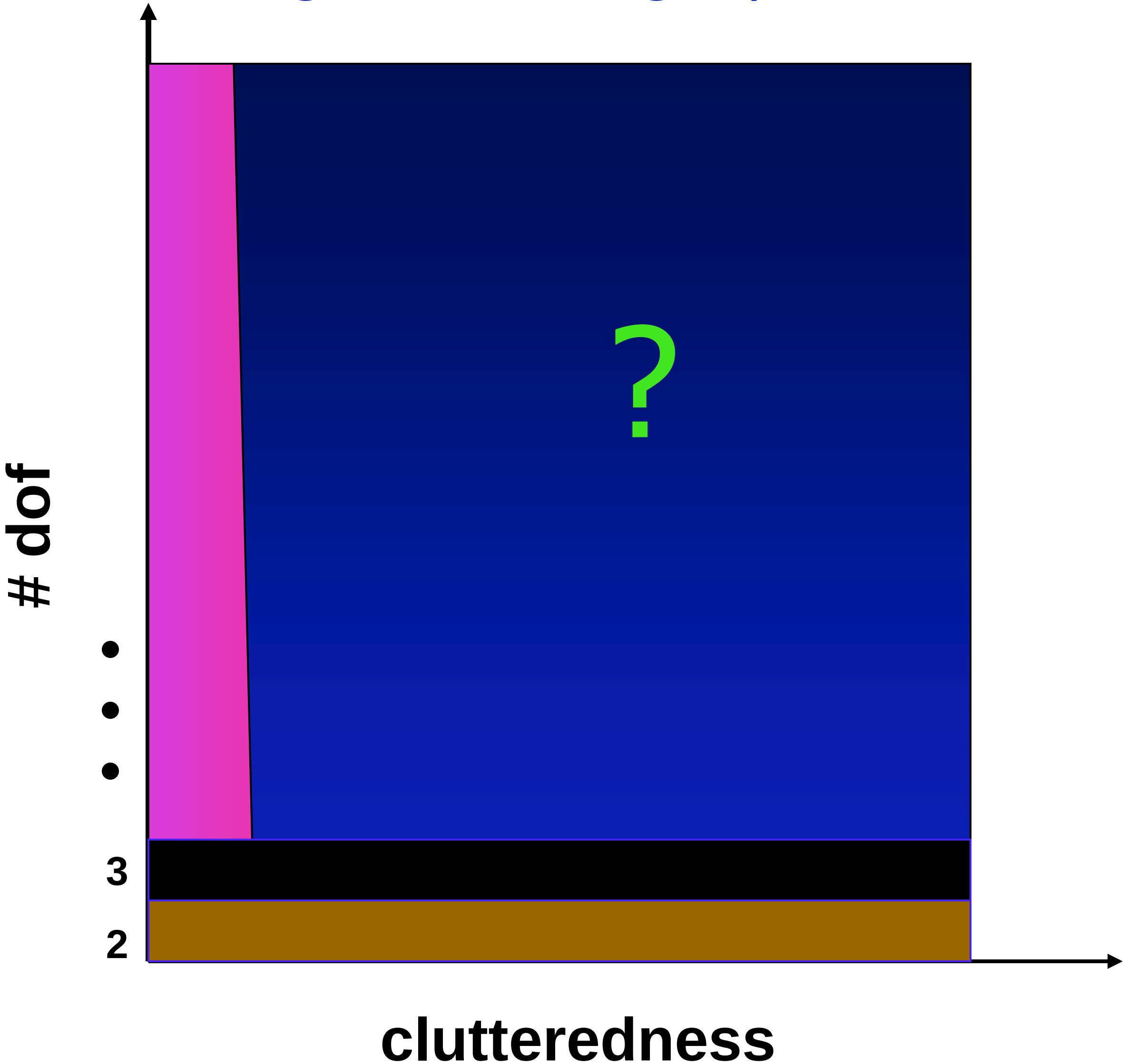




Assembly planning, challenges

- partitioning and sequencing with more complex motion types
- tolerancing, sensitivity analysis
- optimization

The intermediate challenge (narrow passages, not tight)

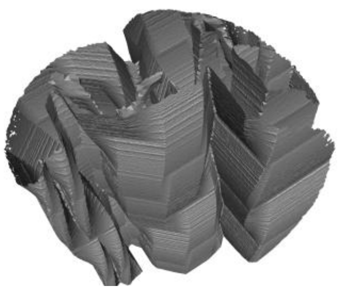
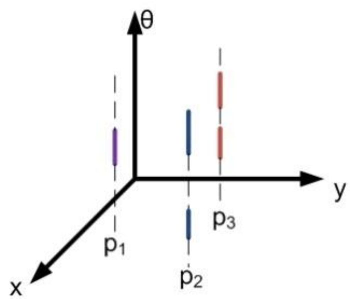


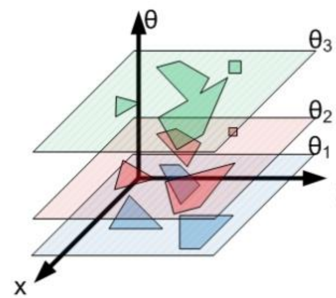
Motion planning via manifold samples

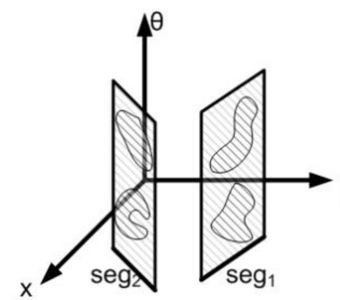
[Salzman-Hemmer-Raveh-Halperin]

Example: polygon translating and rotations among polygons

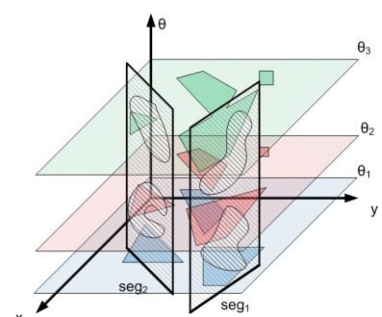
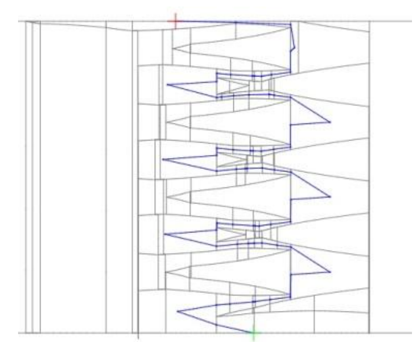
- sampling the 3D configuration space by strong geometric primitives, including exact arrangements of curves



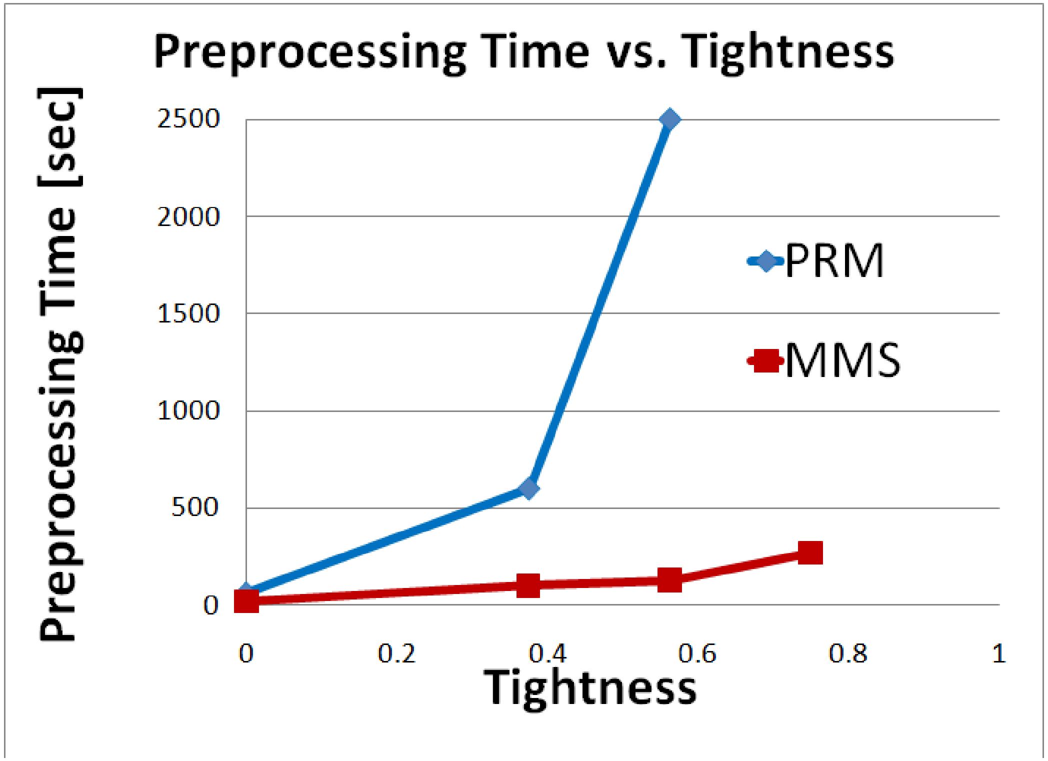


- combinatorial analysis of primitives yields *free space cells*
- path planning by intersecting free space cells

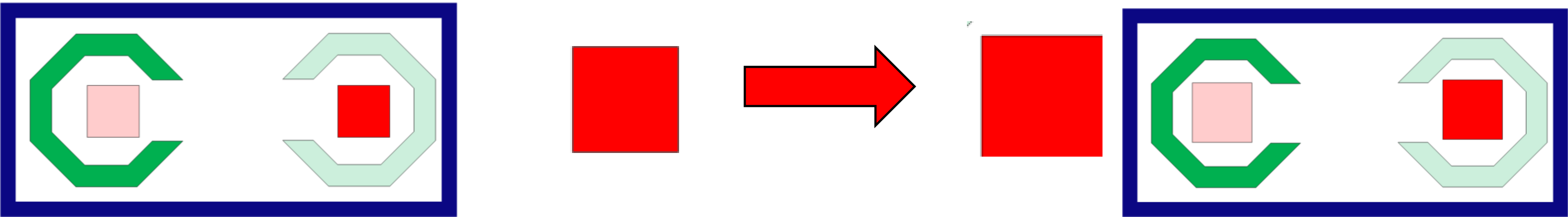



Experimental results (6D C-Space)

- Tightening the configuration space

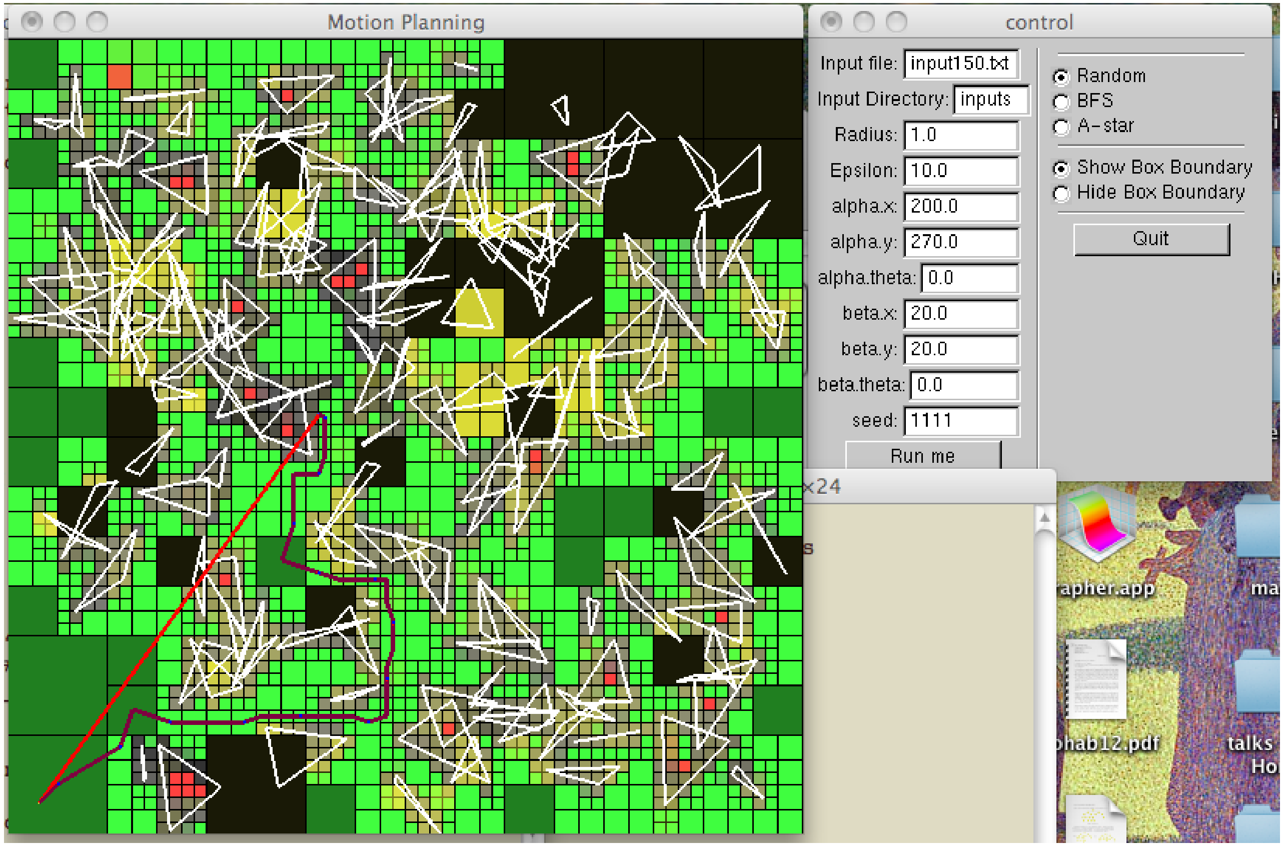


20-fold speedup



Resolution exact (subdivision revisited)

[Yap, Chiang et al]



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