FoDA L5

Bayesian Inference

Bayes' Rule

event M, D

$$\Pr(M \mid D) = \frac{\Pr(D \mid M) \cdot \Pr(M)}{\Pr(D)}$$

$$\underset{\text{proportional to}}{\propto} \Pr(D \mid M) \cdot \Pr(M)$$

$$\Pr(M \mid D) = c \cdot \Pr(D \mid M) \cdot \Pr(M)$$

possibly unknown but fixed constant c.

$$\underbrace{p(M \mid D)}_{\text{posterior}} \propto \underbrace{f(D \mid M)}_{L(M)\text{ likelihood}} \cdot \underbrace{\pi(M)}_{\text{prior } \pi}$$

$\arg\max_M \to$ MAP estimate

$\arg\max_M \to$ MLE

Average Height

Given $D = \{x_1, x_2 \ldots x_n\} = \{1, 3, 5, 9, 12\}$

estimate height μ of typical student at U of U.

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prior $r(\mu) = N_{66,6}(\mu) = \frac{1}{\sqrt{\pi 72}} \exp\left(\frac{-(\mu - 66)^2}{2 \cdot 6^2} \right)$

mean $\mu = 66$

std-dev $\sigma = 6$

inches

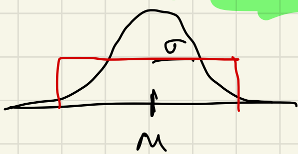
MLE (from likelihood of data D)

$= 5.5$ feet

likelihood $f(D|\mu) = \prod_{x\in D} g(x) = \prod_{x\in D} N_{\mu,2}(x)$

independence

assume $\sigma = 2$



$$= \prod_{x\in D}\left(\frac{1}{\sqrt{8\pi}} \cdot \exp\left(-\frac{1}{8}(\mu-x)^2\right)\right)$$

$\sigma = 6$

$\sigma = 0.1$

$$r(\mu) = \underset{0.02}{\frac{1}{\sqrt{72\pi}}} \exp\left(-\frac{(\mu-66)^2}{\underset{0.02}{72}}\right)$$

posterior

$$p(\mu|D) \propto f(D|\mu)\cdot r(\mu)$$

$$\ln(p(\mu|D)) \propto \ln(f(D|\mu)) + \ln(r(\mu)) + C$$

$$\propto \left(\sum_{x\in D}\left(-\frac{1}{8}(\mu-x)^2\right)\right) - \overset{50}{\cancel{\frac{1}{72}}}(\mu-66)^2 + C'$$

$$\propto -\left(\sum_{x\in D} \cancel{9}(\mu-x)^2\right) - \underset{400}{(\mu-66)^2} + C''$$

Weighted Average

(x_i, w_i)

n values $x_1, x_2, \dots x_n$

n weights $w_1, w_2, \dots w_n$

weight

$w_i \geq 0$

$$\bar{x} = \frac{\sum_{i=1}^{n} w_i \cdot x_i}{\sum_{i=1}^{n} w_i}$$

$$W = \sum_{i=1}^{n} w_i$$

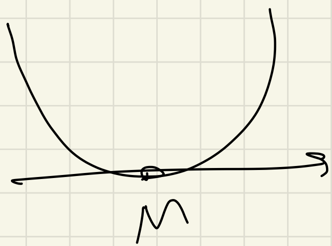
$$p_i = \frac{w_i}{W} \in [0, 1]$$

$$E[x_i] = \sum_{i=1}^{n} p_i \cdot x_i$$

$$\sum_{i=1}^{n} p_i = 1$$

n values $X = \{x_1, x_2, \dots x_n\}$

$$S(M) = \sum_{i=1}^{n} (x_i - M)^2 = \sum_{i=1}^{n} M^2 - 2x_i M - x_i^2$$



$$= nM^2 - \left(2\sum_{i=1}^{n} x_i\right)M + \sum_{i=1}^{n} x_i^2$$

$$\frac{\partial S(M)}{\partial M} = 2nM - 2\sum_{i=1}^{n} x_i = 0$$

$$M = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$p(M \mid D) \propto \exp\left(\ln\left(p(M \mid D) \pm c\right)\right)$$

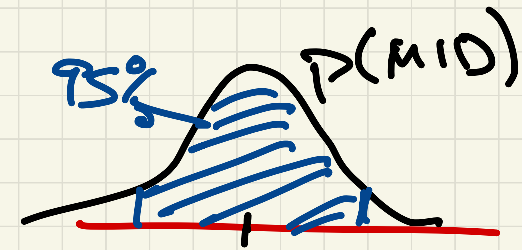
- compare models

  M_1, M_2 $\qquad p(M_1 \mid D) > p(M_2 \mid D)$

$$\frac{p(M_1 \mid D)}{p(M_2 \mid D)} = 2$$

- Marginalize over models

$$\frac{\sum_{M \in \mathcal{M}} p(M \mid D) \cdot h(M)\, \partial M}{\sum_{M \in \mathcal{M}} p(M \mid D)}$$



$M^* = \underset{M}{\arg\max}\ p(M \mid D)$