FoDA L8

Linear Algebra Review #1

Vectors, Matrices: Add, Multiply

Office Hours ?

Move Haocheng Zoom

Office Hour

Monday 7-9pm ?

→ ?

Thursday 3-5pm

Vectors and Matrices

$$(7, 3, 6.3, 7.8)$$

$$v = (v_1, v_2, \dots, v_d) \in \mathbb{R}^d$$

$$v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_d \end{bmatrix} \qquad v^T = \begin{bmatrix} v_1 & v_2 & \dots & v_d \end{bmatrix}$$

Matrix A $\quad n \times d$ matrix $\quad A \in \mathbb{R}^{n \times d}$

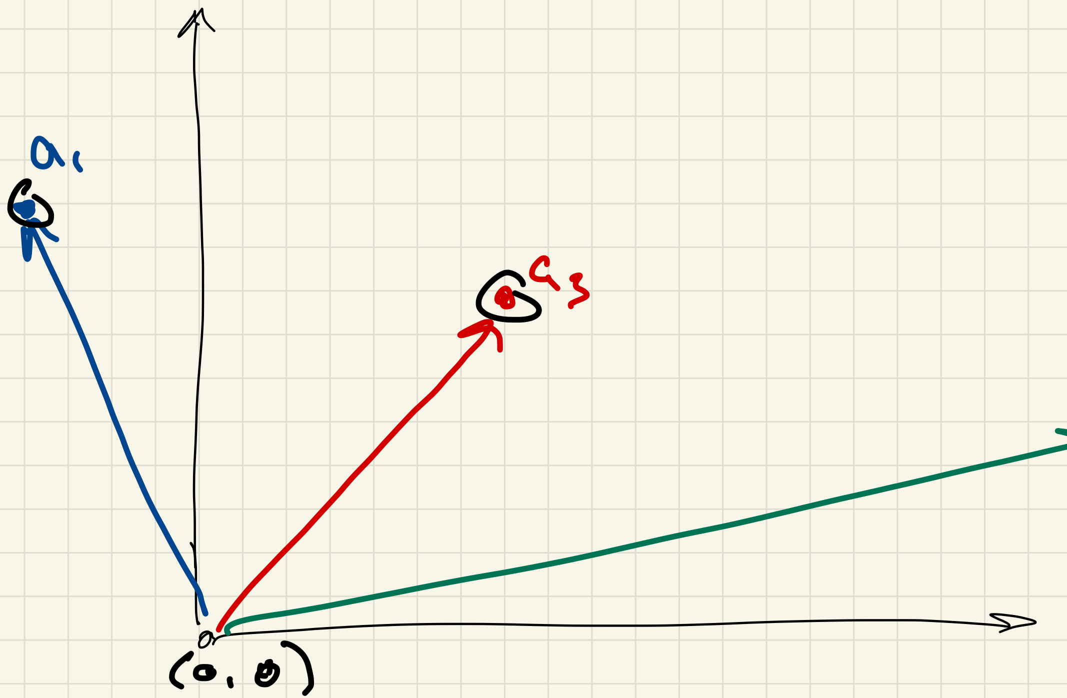
$$A = \begin{bmatrix} A_{1,1} & A_{1,2} & \cdots & A_{1,d} \\ A_{2,1} & A_{2,2} & \cdots & A_{2,d} \\ \vdots & & \ddots & \vdots \\ A_{n,1} & & \cdots & A_{n,d} \end{bmatrix}$$

n rows

d columns

$$= \begin{bmatrix} a_1 \\ a_2 \\ \\ a_n \end{bmatrix} \qquad a_i \in \mathbb{R}^d$$

$\mathbb{R}^2$



$$A = \begin{array}{c} \begin{bmatrix} -0.5 & 1.5 \\ 2.5 & 0.75 \\ 1 & 1 \end{bmatrix} \end{array} \begin{array}{l} a_1 \\ a_2 \\ a_3 \end{array}$$

Transpose

$$A^T = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{bmatrix}$$

$$[A^T]_{ij} = A_{ji}$$

columns

$$A \in \mathbb{R}^{n \times d} \qquad A^T \in \mathbb{R}^{d \times n}$$

rows

Linear Equations

$A \in \mathbb{R}^{2 \times 3}$

$$3x_1 - 7x_2 + 2x_3 = -2$$
$$-1x_1 + 2x_2 - 5x_3 = 6$$

$$A = \begin{bmatrix} 3 & -7 & 2 \\ -1 & 2 & -5 \end{bmatrix} \qquad b = \begin{bmatrix} -2 \\ 6 \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

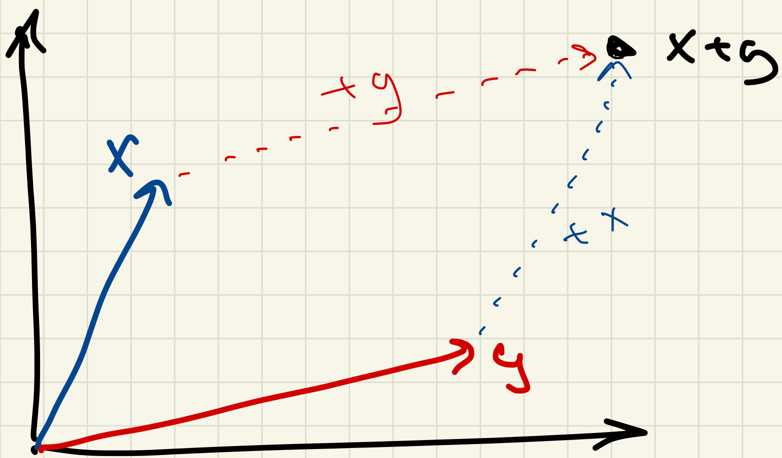
$$Ax = b$$

Addition

"element-wise"

$x, y \in \mathbb{R}^d$ $\quad x = (x_1, x_2, \dots, x_d) \quad y = (y_1, y_2, \dots, y_d)$

$$z = x + y = (x_1 + y_1, x_2 + y_2, \dots, x_d + y_d) \in \mathbb{R}^d$$



$$A, B \in \mathbb{R}^{n \times d}$$

$$C = A + B \qquad C_{ij} = A_{ij} + B_{ij}$$

$i \in [1 \dots n]$

$j \in [1 \dots d]$

$$A = \begin{bmatrix} 3 & -7 & 2 \\ -1 & 2 & -5 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 5 & 3 \\ 4 & 8 & 1 \end{bmatrix}$$

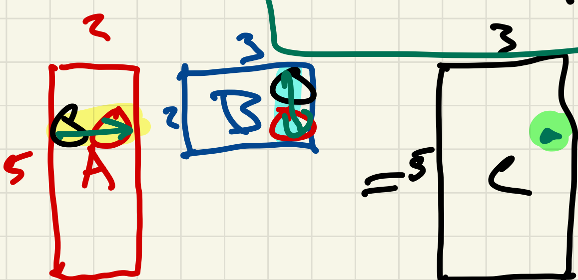
$$A + B = \begin{bmatrix} 5 & -2 & 5 \\ 3 & 10 & -4 \end{bmatrix}$$

Multiplication

$A \in \mathbb{R}^{n \times d}$ $\quad$ $B \in \mathbb{R}^{d \times m}$

$C = AB$ $\quad$ $C \in \mathbb{R}^{n \times m}$

$$\boxed{C_{ij} = \sum_{k=1}^{d} A_{ik} B_{kj}}$$



$C_{2,3}$

Vector-Vector Products

- inner product , dot product

2 column vectors $x, y \in \mathbb{R}^d$

$x \in \mathbb{R}^{d \times 1}$ $y \in \mathbb{R}^{d \times 1}$

$$x^T y = x \cdot y = \langle x, y \rangle = \begin{bmatrix} x_1 & \dots & x_d \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_d \end{bmatrix}$$

$$= \sum_{i=1}^{d} x_i y_i$$

scalar $\alpha \in \mathbb{R}$ $x, y, z \in \mathbb{R}^d$

$$\langle \alpha x, y+z \rangle = \alpha \langle x, y+z \rangle = \alpha \left(\langle x, y \rangle + \langle x, z \rangle \right)$$

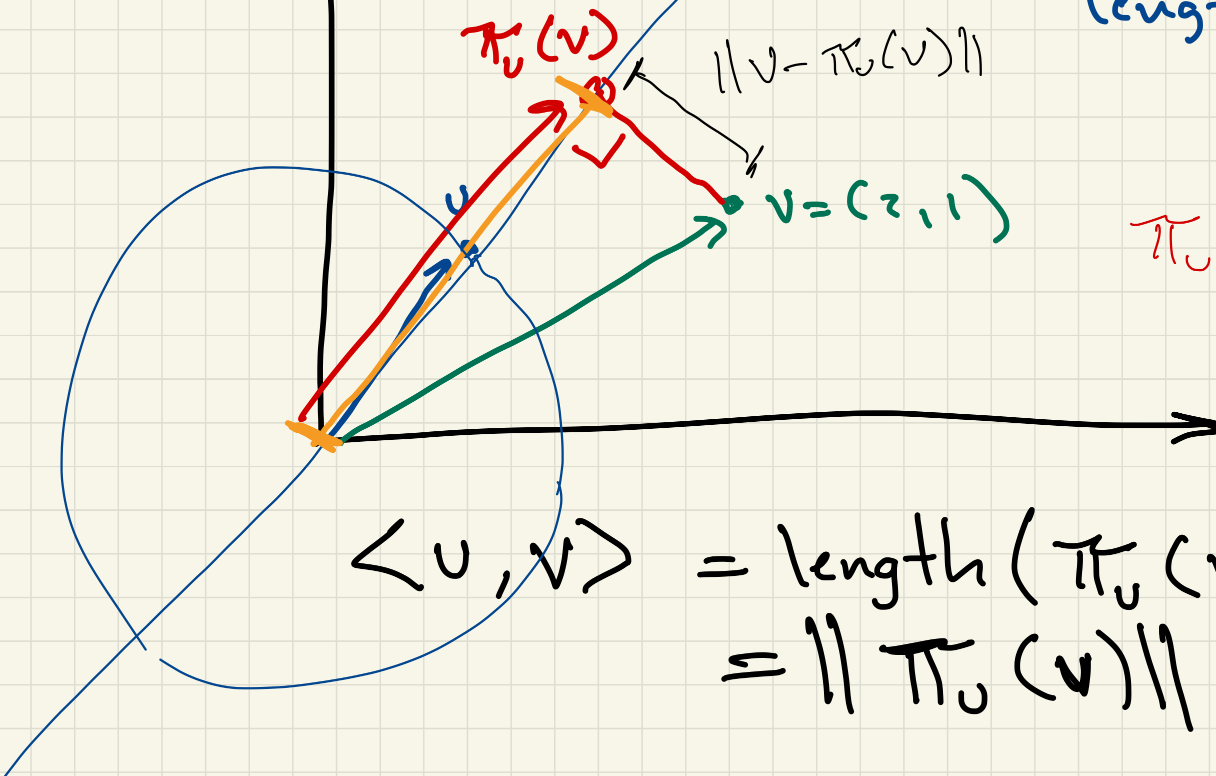
if $\|u\| \neq 1$

$$\langle v, u \rangle \neq \frac{\text{length}(\pi_u(v))}{\text{length}(u)}$$

$$\boxed{u = \left(\frac{3}{5}, \frac{4}{5}\right)}$$

$$v = (2, 1)$$

$$\text{length}(u) = 1$$



$$\pi_u(v) = u \cdot \|\pi_u(v)\|$$

$$= u \cdot \langle u, v \rangle$$

$$\langle u, v \rangle = \text{length}(\pi_u(v))$$

$$= \|\pi_u(v)\|$$

Outer Product

column
$x \in \mathbb{R}^d$
$y \in \mathbb{R}^m$

$$x y^T = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} \begin{bmatrix} y_1 & \cdots & y_m \end{bmatrix}$$

$$= \begin{bmatrix} x_1 y_1 & \cdots & x_1 y_m \\ \vdots & x_i y_j & \vdots \\ x_d y_1 & \cdots & x_d y_m \end{bmatrix}$$

$x \in \mathbb{R}^{d \times 1}$ $\quad$ $y^T \in \mathbb{R}^{1 \times m}$

A $\quad$ B

$C = AB \in \mathbb{R}^{d \times m}$

Matrix-Vector Multiplication

$$A \in \mathbb{R}^{n \times d} \qquad x \in \mathbb{R}^{d}$$

$$y = Ax = \begin{bmatrix} - \; a_1 \; - \\ - \; a_2 \; - \\ \vdots \\ - \; a_n \; - \end{bmatrix} \begin{bmatrix} \\ x \\ \\ \end{bmatrix} = \begin{bmatrix} \langle a_1, x \rangle \\ \langle a_2, x \rangle \\ \vdots \\ \langle a_n, x \rangle \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

matrices: capital letters A, B, C, X

$\mathbb{R}^{n \times d}$

vectors : lower case letters

$\mathbb{R}^d$

u, v, a, b, x, y

scalar : γ, λ

$\mathbb{R}$

latex v