Prob Stats L09a

Joint Probabilities for Discrete R.V.

February 28, 2023

Exponential Distribution

pdf $f(x) = \lambda e^{-\lambda x}$

cdf $F(a) = 1 - e^{-\lambda a}$

$X \sim \mathrm{Exp}(\lambda)$

λ = rate

ex. # events / second

= 1 per second

= 60 per minute

$Pr(X > t)$

$= \sum_{i}^{\infty} (1-p)^{t} [(1-p)^{j}] p$

$= (1-p)^{t} \sum_{j=0}^{\infty} (1-p)^{j} p$

1

Geometric $X \sim \mathrm{Geo}(p)$

n discrete chunks

$f_X(n) = (1-p)^{n} p$

$p = \frac{\lambda t}{n}$

$Pr(X > t)$ $\lim_{n \to \infty} \left(1 - \frac{\lambda t}{n}\right)^{n} \approx e^{-\lambda t} = 1 - F(t)$

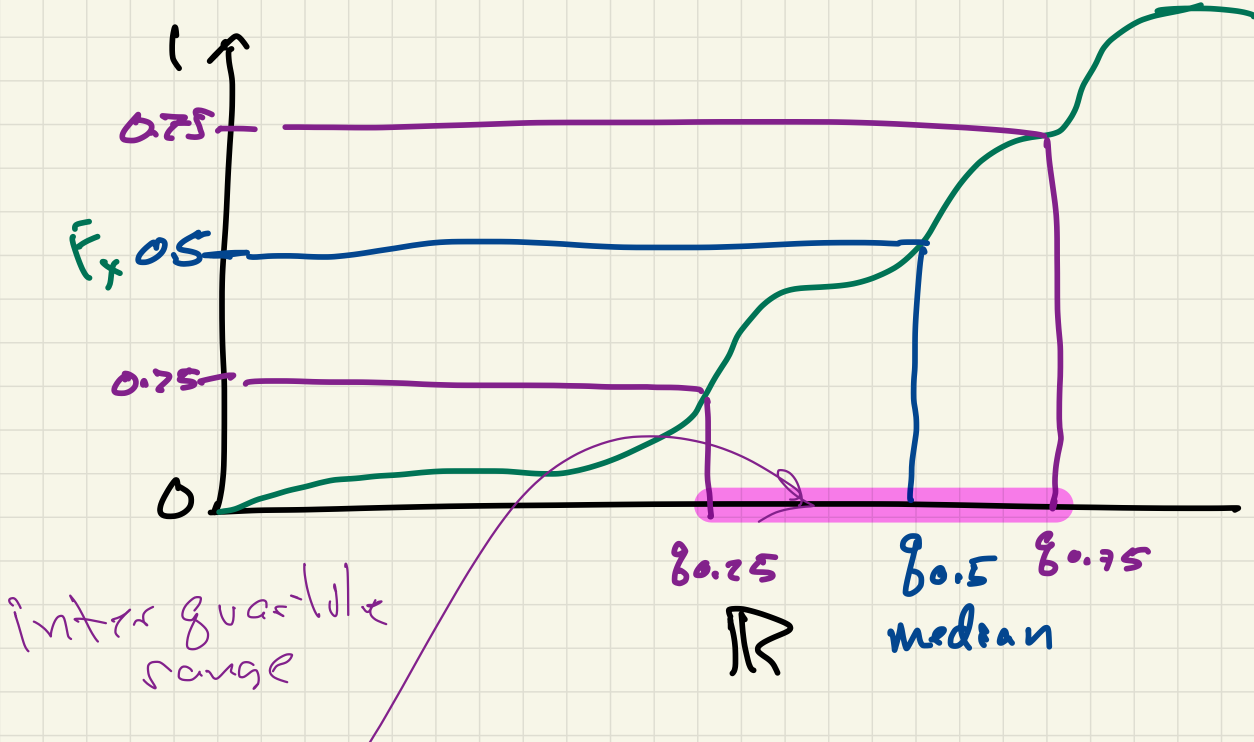
Quantiles

$X \sim \text{Dist}(\theta)$

cdf F_X

pth quantile smallest number q_p s.t.

$$F(q_p) = P_r(X \le q_p) = p$$



Joint Probability for Discrete R.V.

Joint Probability Density Function

2 Random Variables X, Y

event $\{X = a\}$ event $\{Y = b\}$

$$\Pr(\{X=a\} \cap \{Y=b\}) = \Pr(X=a,\ Y=b)$$

$$= f_{X,Y}(a,b) \leftarrow \text{pdf}$$

2 RV X, Y

pdf $f_{X,Y}(a,b) = Pr(X=a, Y=b)$

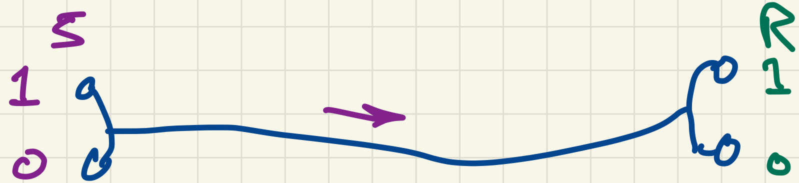
① $f_{X,Y}(a,b) \in [0,1]$ for all a, b

② $\sum_i \sum_j f_{X,Y}(a_i, b_j) = 1$ $\forall$ outcome a_i, b_j

$a_i \neq a_{i'}$

$b_j \neq b_{j'}$

Example



marginal probability

$f_S(0) = 0.53$

		R = 0	R = 1	
S	0	0.45	0.08	$0.53 = 0.45 + 0.08$
	1	0.06	0.41	$0.47 = f_S(1)$

$f_R(0) = 0.51 + f_R(1) = 0.49 = 1.00$

$f_{S,R}(0,0) = 0.45$

Marginal probability

$$P(X=a) = \sum_j \Pr(X=a, Y=b_j) = \sum_j f_{X,Y}(a, b_j)$$

2 fair coins

$\Omega = \{hh, ht, th, tt\}$

2 R.V. $H = \#\text{ heads} \in \{0, 1, 2\}$

$B =$ value in binary $h = 0 \quad t = 1$

$hh = 0$ (00) $\quad ht = 1$ (01) $\quad th = 2$ (10) $\quad tt = 3$ (11)

B

$f_{H,B} =$

H \ B	00	01	10	11	$P(H=i)$
0	0	0	0	1/4	$= 0$: 1/4
1	0	1/4	1/4	0	$= 1$: 1/2
2	1/4	0	0	0	$= 2$: $= 1/4$

f_H

f_B:

00	01	10	11
1/4	1/4	1/4	1/4

Conditional Probability

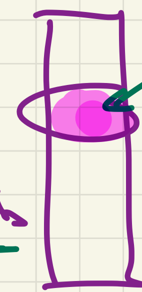
2 RVs X, Y

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\Pr(X = a \mid Y = b) = \frac{\Pr(X = a,\ Y = b)}{\Pr(Y = b)}$$

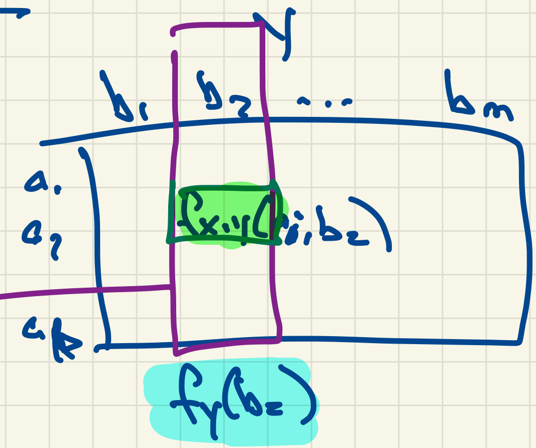
$$f_{X \mid Y=b}(a) = \frac{f_{X,Y}(a, b)}{f_Y(b)}$$

$f_{X \mid Y=b_2}$



$$\frac{f_{X,Y}(a_i, b_2)}{f_Y(b_2)}$$

X



R \ S	0	1
0	0.45	0.08
1	0.06	0.41
	0.51	0.49

$f_{R \mid S=1}$

0	1
$\frac{0.08}{0.49}$ +	$\frac{0.41}{0.49}$

$\Rightarrow \frac{0.08+0.4}{0.49}$