Noname manuscript No.
(will be inserted by the editor)

Dynamic Monitoring of Optimal Locations in Road Network Databases

Bin Yao[1], Xiaokui Xiao[2], Feifei Li[3], Yifan Wu[1],
[1]***Department of Computer Science and Engineering, Shanghai Key Laboratory of Scalable Computing and Systems, Shanghai Jiao Tong University, Shanghai, China***
[2]***School of Computer Engineering, Nanyang Technological University, Singapore***
[3]***School of Computing, University of Utah***
[1]***yaobin@cs.sjtu.edu.cn, wuyf@sjtu.edu.cn,*** [2]***xkxiao@ntu.edu.sg,*** [3]***lifeifei@cs.utah.edu***

the date of receipt and acceptance should be inserted later

Abstract Optimal location (OL) queries are a type of spatial queries that are particularly useful for the strategic planning of resources. Given a set of existing facilities and a set of clients, an OL query asks for a location to build a new facility that optimizes a certain cost metric (defined based on the distances between the clients and the facilities). Several techniques have been proposed to address OL queries, assuming that *all clients and facilities reside in an L_p space*. In practice, however, movements between spatial locations are usually confined by the underlying road network, and hence, the actual distance between two locations can differ significantly from their L_p distance.

Motivated by the deficiency of the existing techniques, this paper presents a comprehensive study on OL queries in road networks. We propose a unified framework that addresses three variants of OL queries that find important applications in practice, and we instantiate the framework with several novel query processing algorithms. We further extend our framework to efficiently monitor the OLs when locations for facilities and/or clients have been updated. Our dynamic update methods lead to efficient answering of continuous optimal location queries. We demonstrate the efficiency of our solutions through extensive experiments with large real data.

1 Introduction

An *optimal location (OL) query* concerns three spatial point sets: a set F of *facilities*, a set C of *clients*, and a set P of *candidate locations*. The objective of this query is to identify a candidate location $p \in P$, such that a new facility built at p can optimize a certain cost metric that is defined based on the distances between the facilities and the clients. OL queries find important applications in the strategic planning of resources (e.g., hospitals, post offices, banks, retail facilities) in both public and private sectors [2, 6, 29]. As an example, we illustrate three OL queries based on different cost metrics.

Example 1 Julie would like to open a new supermarket in San Francisco that can attract as many customers as possible. Given the set F (C) of all existing supermarkets (residential locations) in the city, Julie may look for a candidate location p, such that a new supermarket on p would be the closest supermarket for the largest number of residential locations. ■

Example 2 John owns a set F of pizza shops that deliver to a set C of places in Gotham city. In case that John wants to extend his business by adding another pizza shop, a natural choice for him is a candidate location that minimizes the average distance from the points in C to their respective nearest pizza shops. ■

Example 3 Gotham city government plans to establish a new fire station. Given the set F (C) of existing fire stations (buildings), the government may seek a candidate location that minimizes the maximum distance from any building to its nearest fire station. ■

Several techniques [2, 6, 26, 29] have been proposed for processing OL queries under various cost metrics. All those techniques, however, assume that F and C are point sets in an L_p space. This assumption is rather restrictive because, in practice, movements between spatial locations are usually confined by the underlying road network, and hence, the commute distance between two locations can differ significantly from their L_p distance. Consequently, the existing solutions for OL queries cannot provide useful results for practical applications in road networks.

Address(es) of author(s) should be given

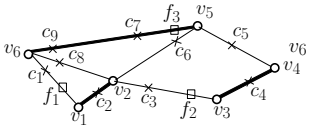


Fig. 1 Example of G^o

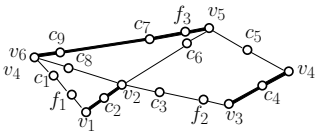


Fig. 2 Example of G

Problem Formulation. This paper presents a novel and comprehensive study on OL queries in road network databases. We consider a problem setting as follows. First, any facility in F or any client in C should locate on an edge in an undirected connected graph $G^\circ = (V^\circ, E^\circ)$, where V° (E°) denotes the set of vertices (edges) in G°. Second, every client $c \in C$ is associated with a positive weight $w(c)$ that captures the importance of the client. For example, if each client point c represents a residential location, then $w(c)$ may be specified as the size of the population residing at c. Third, there should exist a user-specified set E_c° of edges in E°, such that a new facility f can be built on any point on any edge in E_c°, as long as f does not overlap with an existing facility in F. E_c° can be arbitrary, e.g., we can have $E_c^\circ = E^\circ$. We define P as the set of points on the edges in E_c° that are not in F, and we refer to any point in P as a *candidate location*. For example, Figure 1 illustrates a road network that consists of 5 vertices and 8 edges. The squares (crosses) in the Figure denote the facilities (clients) in the road network. The highlighted edges are the user-specified set E_c° of edges where a new facility may be built.

We investigate three variants of OL queries as follows:

1) The *competitive location query* asks for a candidate location $p \in P$ that maximizes the total weight of the clients *attracted* by a new facility built on p. Specifically, we say that a client c is *attracted* by a facility f, and that f is an *attractor* for c, if the network distance $d(c, f)$ between c and f is at most the distance between c and any facility in F. In other words, the competitive location query ensures that

$$p = \operatorname*{argmax}_{p \in P} \sum_{c \in C_p} w(c), \tag{1}$$

where $C_p = \{c \mid c \in C \ \wedge \ \forall f \in F, d(c,p) \leq d(c,f)\}$, i.e., C_p is the set of clients attracted by p. Example 1 demonstrates an instance of this query.

2) The *MinSum location query* asks for a candidate location $p \in P$ on which a new facility can be built to minimize the total *weighted attractor distance* (WAD) of the clients. In particular, the WAD of a client c is defined as $\widehat{a}(c) = w(c) \cdot a(c)$, where $a(c)$ denotes the distance from c to its attractor (referred as the *attractor distance* of c). That is, the MinSum location query requires that

$$\begin{aligned} p &= \operatorname*{argmin}_{p \in P} \sum_{c \in C} w(c) \cdot \min\left\{d(c,f) \mid f \in F \cup \{p\}\right\} \\ &= \operatorname*{argmin}_{p \in P} \sum_{c \in C} \widehat{a}(c). \end{aligned} \tag{2}$$

Example 2 shows a special case of the MinSum location query where all clients have the same weight.

3) The *MinMax location query* asks for a candidate location $p \in P$ to construct a new facility that minimizes the maximum WAD of the clients, i.e.,

$$p = \operatorname*{argmin}_{p \in P} \big(\max_{c \in C} \left\{\widehat{a}(c) \mid F = F \cup \{p\}\right\}\big). \tag{3}$$

Example 3 illustrates a MinMax location query.

One fundamental challenge in answering an OL query is that there exists an *infinite* number of candidate locations in P where the new facility may be built, i.e., P is a continuous domain on the edges of the network. (Recall that P contains *all* points on the edges in the user-specified set E_c°, except the points where existing facilities are located.) This necessitates query processing techniques that can identify query results without enumerating all candidate locations. Another complicating issue is that the answer to an OL may not be unique, i.e., there may exist multiple candidate locations in P that satisfy Equation 1, 2, or 3. We propose to identify *all* answers for any given optimal location query, and return them to the user for final selection. This renders the problem even more challenging, since it requires additional efforts to ensure the completeness of the query results.

Lastly, in some applications it is common that clients or existing facilities have moved on the road network after the last execution of OL queries. Instead of computing the OLs again from scratch, we expect to monitor the query results in an incremental fashion, which may dramatically reduce the query cost (compared to the naive approach of recomputing the OLs after location updates for facilities and clients).

Contributions. In this paper, we propose a unified solution that addresses all aforementioned variants of optimal location queries in road network databases. Our first contribution is a solution framework based on the divide-and-conquer paradigm. In this framework, we process a query by first (i) dividing the edges in G° into smaller intervals, then (ii) computing the best query answers on each interval, and finally (iii) combining the answers from individual intervals to derive the global optimal locations. A distinct feature of this framework is that most of its algorithmic components are *generic*, i.e., they are not specific to any of the three types of OL queries. This significantly simplifies the design of query processing algorithms, and enables us to develop general optimization techniques that work for all three query types.

Secondly, we instantiate the proposed framework with a set of novel algorithms that optimize query efficiency by exploiting the characteristics of OL queries. We provide theoretical analysis on the performance of each algorithm in terms of time complexity and space consumption.

Thirdly, we present extensions to our framework to enable the incremental monitoring of the query results from OL queries, when the locations of facilitates or clients have changed.

Last, we demonstrate the efficiency of our algorithms with extensive experiments on large-scale real datasets. In particular, our algorithms can answer an OL query efficiently on a commodity machine, in a road network with 174,955 vertices and 500,000 clients. Furthermore, the query result can be incrementally updated in just a few seconds after a location update for either a client or a facility.

2 Related Work

The problem of locating "preferred" facilities with respect to a given set of client points, referred to as the *facility location problem*, has been extensively studied in past years (see [8,20] for surveys). In its most common form, the problem (i) involves a finite set C of clients and a finite set P of candidate facilities, and (ii) asks for a subset of k ($k > 0$) facilities in P that optimizes a predefined metric. The problem is polynomial-time solvable when k is a constant, but is NP-hard for general k [8, 20]. Furthermore, existing solutions do not scale well for large P and C. Hence, existing work on the problem mainly focuses on developing approximate solutions.

OL queries can be regarded as variations of the facility location problem with three modified assumptions: (i) P is an infinite set, (ii) $k = 1$, i.e., only *one* location in P is to be selected (but all locations that tie with each other need to be returned), and (iii) a finite set F of facilities has been constructed in advance. These modified assumptions distinguish OL queries from the facility location problem.

Previous work [2, 6, 26, 29] on OL queries considers the case when the transportation cost between a facility and a client is decided by their L_p distance. Specifically, Cabello et al. [2] and Wong et al. [26] investigate competitive location queries in the L_2 space. Du et al. [6] and Zhang et al. [29] focus on the L_1 space, and propose solutions for competitive and MinSum location queries, respectively. None of the solutions developed therein is applicable when the facilities and clients reside in a road network.

There also exist two other variations of the facility location problem, namely, the *single facility location* problem [8, 20] and the *online facility location* problem [9, 17], that are related to (but different from) OL queries. The single facility location problem asks for *one* location in P that optimizes a predefined metric with respect to a given set C of clients. It requires that no facility has been built previously, whereas OL queries consider the existence of a set F of facilities.

The online facility location problem assumes a dynamic setting where (i) the set C of clients is initially empty, and (ii) new clients may be inserted into C as time evolves. It asks for a solution that constructs facilities incrementally (i.e., one at a time), such that the quality of the solution (with respect to some predefined metric) is competitive against any solutions that are given all client points in advance. This problem is similar to OL queries, in the sense that they all aim to optimize the locations of new facilities based on the existing facilities and clients. *However, the techniques [9, 17] for the online facility location problem cannot address OL queries*, since those techniques assume that the set P of candidate facility locations is finite; in contrast, OL queries assume that P contains an infinite number of points, e.g., P may consist of all points (i) in an L_p space (as in [2,6,26,29]) or (ii) on a set of edges in a road network (as in our setting).

In the preliminary version of this paper [27], we investigated the static version of optimal location queries. Compared with the preliminary version, this paper presents a new study on handling updates in the locations of facilities and clients. In particular, we present novel incremental methods to identify OLs after updates for all three types of optimal location queries. We also include an extensive experiments that demonstrate the efficiency of the incremental update methods over the naive approach of recomputing OLs from scratch after each update (using the static methods from our preliminary version [27]).

Ghaemi et al. [10–12] studied static and dynamic versions of competitive location queries. Their solutions, however, are not applicable for MinSum and MinMax location queries. In contrast, we present a uniform framework for all three variants of optimal location queries. Furthermore, as will be shown in Section 10, our solution for competitive location queries has a much lower memory consumption than Ghaemi et al.'s while only incurring a slightly higher computation cost.

Lastly, there is a large body of literature on query processing techniques for road network databases [3, 4, 15, 16, 19, 21–24, 28]. Most of those techniques are designed for the *nearest neighbor* (NN) query [16, 21, 22] or its variants, e.g., *approximate NN queries* [23, 24], *aggregate NN queries* [28], *continuous NN queries* [19], *path NN queries* [3], etc. None of those techniques can address the problem we consider, due to the fundamental differences between NN queries and OL queries. Such differences are also demonstrated by the fact that, despite the plethora of solutions for L_p-space NN queries, considerable research effort [2, 6, 26, 29] is still devoted to OL queries in L_p spaces.

3 Solution Overview

We propose one unified framework for the three variants of OL queries. In a nutshell, our solution adopts a divide-and-conquer paradigm as follows. First, we divide the edges in E° into smaller intervals, such that all facilities and clients fall on only the endpoints (but not the interior) of the intervals. As a second step, we collect the intervals that are segments of some edges in E_c°, i.e, all points in such an interval are candidate locations in P. Then, we traverse those intervals in a certain order. For each interval I examined, we compute the *local* optimal locations on I, i.e., the points on I that provide a better solution to the OL query than any other points on I. The *global* optimal locations are pinpointed and returned, once we confirm that none of the unvisited intervals can provide a better solution than the best local optima found so far.

In the following, we will introduce the basic idea of each step in our framework; the details of our algorithms will be presented in Sections 4-8. For convenience, we define n as the maximum number of elements in V°, E°, C, and F, i.e., $n = \max\{|V^\circ|, |E^\circ|, |C|, |F|\}$. Table 1 summarizes the notations frequently used in the paper.

Construction of Road Intervals. We divide the edges in E° into intervals, by inserting all facilities and clients into the road network $G^\circ = (V^\circ, E^\circ)$. Specifically, for each point $\rho \in C \cup F$, we first identify the edge $e \in E^\circ$ on which ρ locates. Let v_l and v_r be the two vertices connected by e. We then break e into two road segments, one from v_l to ρ and the other from ρ to v_r. As such, ρ becomes a vertex in the network. Once all facilities and clients have been inserted into G°, we obtain a new road network $G = (V, E)$ where $V = V^\circ \cup C \cup F$. For example, Figure 2 illustrates a road network transformed from the one in Figure 1. Transforming G° to G requires only $O(n)$ space and $O(n)$ time, since $|C| = O(n)$, $|F| = O(n)$, and it takes only $O(1)$ time to add a vertex in G°. In the sequel, we simply refer to G as our road network.

Traversal of Road Intervals. After G is constructed, we collect the set E_c of edges in E that are partial segments of some edges in E_c°. For example, the highlighted edges in Figure 2 illustrate the set E_c that correspond to the set E_c° of highlighted edges in Figure 1. As a next step, we traverse E_c to look for the optimal locations. A straightforward approach is to process the edges in E_c in a random order, which, however, incurs significant overhead, since the optimal locations cannot be identified until all edges in E_c are inspected. Section 6 addresses this issue with novel techniques that avoid the exhaustive search on E_c. The idea is to first divide E_c into subsets, and then process the subsets in descending order of their likelihood of containing the optimal locations.

Table 1 Frequently Used Notations

Symbol	Description
$G^\circ = (V^\circ, E^\circ)$	the road network with vertex (edge) set V° (E°)
C	the set of clients
F	the set of existing facilities
E_c°	the user-specified set of edges on which the new facility can be built
P	the set of candidate locations
$d(p_1, p_2)$	the network distance between points p_1 and p_2
$w(c)$	the weight of a client c
$a(c)$	the attractor distance of a client c
$\widehat{a}(c)$	the weighted attractor distance of a client c
C_p	the set of clients attracted by a point p
n	$n = \max\{\|V^\circ\|, \|E^\circ\|, \|C\|, \|F\|\}$
$G = (V, E)$	the road network transformed from G° (see Section 3)
E_c	the set of edges in E that are segments of the edges in E_c° (see Section 3)
$\mathcal{A}(v)$	the *attraction set* of a vertex v in G (see Section 3)
$m(p)$	the *merit* of a point p (see Section 4.2)

Identification of Local Optimal Locations. In Section 4, we will present algorithms for computing the local optimal locations on any edge $e \in E_c$, based on (i) the attractor distance of each client, and (ii) the *attraction set* $\mathcal{A}(v)$ of each endpoint v of e. Specifically, the attraction set $\mathcal{A}(v)$ contains entries of the form $\langle c, d(c, v)\rangle$, for any client c such that $d(c, v) \leq a(c)$. That is, $\mathcal{A}(v)$ records the clients that are closer to v than to their respective attractors (i.e., the respective nearest facilities). The attraction sets of e's endpoints are crucial to our algorithm, since they capture all clients that might be affected by a new facility built on e (see Section 4 for a detailed discussion). We will present our algorithms for computing attraction sets and attractor distances in Section 5.

Updates of Facilities and Clients. In Sections 7 and 8, we present algorithms for incrementally monitoring the results of OL queries when there are updates in the locations of facilities and/or clients. The basic idea of our algorithms is to (i) maintain auxiliary information about the solutions to OL queries, and (ii) utilize the auxiliary information to accelerate the re-computation of query results in case of updates.

4 Local Optimal Locations

This section presents our initial algorithms for computing local optimal locations on any edge $e \in E_c$, given the attraction sets of e's endpoints, and the attractor distances of the clients. For ease of exposition, we will elaborate our algorithms under the assumption that none of e's endpoints is an existing facility in F, i.e., both endpoints of e are candidate locations in P. We will discuss how our algorithms can be extended (for the general case) in the end of the discussion for each query type.

Algorithm *CompLoc* (e)

1. construct an empty one-dimensional plane R
2. let ℓ be the length of e, and v_l (v_r) be the left (right) endpoint of e
3. for each client c that appears in $\mathcal{A}(v_l)$ but not $\mathcal{A}(v_r)$
4. create in R a line segment $[0, a(c) - d(c, v_l)]$
5. assign a weight $w(c)$ to the segment
6. for each client c that appears in $\mathcal{A}(v_r)$ but not $\mathcal{A}(v_l)$
7. create in R a segment $[\ell - a(c) + d(c, v_r), \ell]$ with a weight $w(c)$
8. for each client c that appears in both $\mathcal{A}(v_l)$ and $\mathcal{A}(v_r)$
9. if $\ell \leq 2 \cdot a(c) - d(c, v_l) - d(c, v_r)$
10. create in R a line segment $[0, \ell]$ with a weight $w(c)$
11. else
12. create in R two line segments $[0, a(c) - d(c, v_l)]$ and $[\ell - a(c) + d(c, v_r), \ell]$, each with a weight $w(c)$
13. compute the intervals $I \subseteq [0, \ell]$, such that I maximizes the total weights of the line segments in R that fully cover I
14. return the intervals identified at Line 13

Fig. 3 The *CompLoc* Algorithm

4.1 Competitive Location Queries

Recall that a competitive location query asks for a new facility that maximizes the total weight of the clients attracted by it. Intuitively, to decide the optimal locations for such a new facility on a given edge $e \in E_c$, it suffices to identify the set of clients that can be attracted by each point p on e. As shown in the following lemma, the clients attracted by any p can be easily computed from the attraction sets of e's endpoints.

Lemma 1 *A client c is attracted by a point p on an edge $e \in E_c$, iff there exists an entry $\langle c, d(c, v)\rangle$ in the attraction set of an endpoint v of e, such that $d(c, v) + d(v, p) \leq a(c)$.*

Proof Observe that $d(c, p) \leq d(c, v) + d(v, p)$. Hence, when $d(c, v) + d(v, p) \leq a(c)$, we have $d(c, p) \leq a(c)$, i.e., c is attracted by p. Thus, the "if" direction of the lemma holds.

Now consider the "only if" direction. Since p is a point on e, the shortest path from p to c must go through an endpoint v of e. Observe that $d(p, c) \geq d(v, c)$. Therefore, if c is attracted by p, we have $a(c) \geq d(p, c) \geq d(v, c)$, which indicates that $\langle c, d(c, v)\rangle$ must be an entry in $\mathcal{A}(v)$.

Based on Lemma 1, we propose the *CompLoc* algorithm (in Figure 3) for finding local competitive locations on an edge $e \in E_c$. We illustrate the algorithm with an example.

Example 4 Suppose that we apply *CompLoc* on an edge e_0 with a length $\ell = 5$. Figure 4(a) illustrates $\mathcal{A}(v_l)$ and $\mathcal{A}(v_r)$, where v_l (v_r) is the left (right) endpoint of e_0. Assume that each client c has a weight $w(c) = 1$ and an attractor distance $a(c) = 5$.

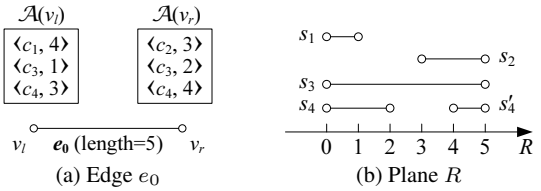
CompLoc starts by creating a one-dimensional plane R. After that, it identifies those clients that appear in $\mathcal{A}(v_l)$ but not $\mathcal{A}(v_r)$. By Lemma 1, for any c of those clients, if c is attracted to a point p on e_0, then $d(p, v_l) \in [0, a(c) - d(c, v_l)]$, and vice versa. To capture this fact, *CompLoc* creates in R a line segment $[0, a(c) - d(c, v_l)]$, and assigns a weight $w(c) = 1$ to the segment. In our example, c_1 is the only client that appears in $\mathcal{A}(v_l)$ but not $\mathcal{A}(v_r)$, and $a(c_1) - d(c_1, v_l) = 1$. Hence, *CompLoc* adds in R a segment $s_1 = [0, 1]$ with a weight $w(c_1) = 1$, as illustrated in Figure 4(b).

Next, *CompLoc* examines the only client c_2 that is contained in $\mathcal{A}(v_r)$ but not $\mathcal{A}(v_l)$. By Lemma 1, a point $p \in e_0$ is an attractor for c, if and only if $d(p, v_l) \in [\ell - a(c_2) + d(c_2, v_r), \ell]$. Accordingly, *CompLoc* inserts in R a segment $s_2 = [\ell - a(c_2) + d(c_2, v_r), \ell]$ with a weight $w(c_2) = 1$.

After that, *CompLoc* identifies the clients c_3 and c_4 that appear in both $\mathcal{A}(v_l)$ and $\mathcal{A}(v_r)$. For c_3, we have $\ell \leq 2 \cdot a(c_3) - d(c_3, v_l) - d(c_3, v_r)$, which (by Lemma 1) indicates that any point on e_0 can attract c_3. Hence, *CompLoc* creates in R a segment $[0, 5]$ with a weight $w(c_3) = 1$. On the other hand, since $\ell > 2 \cdot a(c_4) - d(c_4, v_l) - d(c_4, v_r)$, a point p on e_0 can attract c_4, if and only if $d(p, v_l) \in [0, a(c_4) - d(c_4, v_l)]$ or $d(p, v_l) \in [\ell - a(c_4) + d(c_4, v_r), \ell]$. Therefore, *CompLoc* inserts in R two segments $s_4 = [0, 2]$ and $s'_4 = [4, 5]$, each with a weight 1 (see Figure 4(b)).

As a next step, *CompLoc* scans through the line segments in R to compute the local competitive locations on e_0. Let p be any point on e_0, and o be the point in R whose coordinate equals the distance from p to v_l. Observe that, a client $c \in C$ is attracted by p, if and only if there exists a segment s in R, such that (i) s is constructed from c and (ii) s covers o. Therefore, to identify the local competitive locations on e_0, it suffices to derive the intervals I in R, such that (i) $I \subseteq [0, \ell]$, and (ii) I maximizes the total weight of the line segments that fully cover I. Such intervals can be computed by applying a standard *plane sweep* algorithm [1] on the line segments in R. In our example, the local competitive locations on e_0 correspond to two intervals in R, namely, $[0, 1]$ and $[4, 5]$, each of which is covered by three segments with a total weight 3. Finally, *CompLoc* terminates by returning the two intervals $[0, 1]$ and $[4, 5]$, as well as the weight 3. ■

Our discussion so far assumes that no facility in F locates on an endpoint of the given edge e. Nevertheless, *CompLoc* can be easily extended for the case when either of e's endpoints is a facility. The only modification required is that, we need to exclude the facility endpoint(s) of e, when we construct the line segment(s) on R that corresponds to each client. For example, if we have a line segment $[0, 5]$ and the left endpoint of e is a facility, then we should modify segment as $(0, 5]$ before we compute the local competitive locations on e. The case when the right endpoint of e is a facility can be handled in a similar manner.



(a) Edge e_0 (b) Plane R

Fig. 4 Demonstration of *CompLoc*

CompLoc runs in $O(n \log n)$ time and $O(n)$ space. First, constructing line segments in R takes $O(n)$ time and $O(n)$ space, since (i) there exist $O(n)$ clients in the attraction sets of the endpoints of e, (ii) at most two segments are created from each client. Second, since there are only $O(n)$ line segments in R, the plane sweep algorithm on the segments runs in $O(n \log n)$ time and $O(n)$ space.

4.2 MinSum Location Queries

For any candidate location p, we define the *merit* of p (denoted as $m(p)$) as

$$m(p) = \sum_{c \in C} w(c) \cdot \max\{0, a(c) - d(c,p)\}.$$

That is, $m(p)$ captures how much the total WAD of all clients may reduce, if a new facility is built on p. A point is a local MinSum location on an edge $e \in E_c$, if and only if it has the maximum merit among all points on e. Interestingly, the merit of the points on any edge e is always maximized at one endpoint of e, as shown in the following lemma.

Lemma 2 *For any point p in the interior of an edge $e \in E$, if $m(p)$ is larger than the merit of one endpoint of e, then $m(p)$ must be smaller than the merit of the other endpoint.*

Proof Let v_l (v_r) be the left (right) endpoint of e. Recall that C_p is the set of clients attracted by p. First of all,

$$\begin{aligned} m(v_l) &= \sum_{c \in C} w(c) \cdot \max\{0, a(c) - d(c, v_l)\} \\ &\geq \sum_{c \in C_p} w(c) \cdot \big(a(c) - d(c, v_l)\big), \text{ and similarly,} \\ m(v_r) &\geq \sum_{c \in C_p} w(c) \cdot \big(a(c) - d(c, v_r)\big). \end{aligned} \tag{4}$$

Assume w.l.o.g. that $m(p) > m(v_l)$. We have

$$\begin{aligned} m(p) &= \sum_{c \in C_p} w(c) \cdot \big(a(c) - d(c,p)\big) \\ &\geq m(v_l) \geq \sum_{c \in C_p} w(c) \cdot \big(a(c) - d(c, v_l)\big), \end{aligned}$$

which leads to

$$\sum_{c \in C_p} w(c)(d(c, v_l) - d(c,p)) > 0. \tag{5}$$

Let C_p^l (C_p^r) be the subset of clients c in C_p, such that the shortest path from c to p passes through v_l (v_r). Clearly, $C_p^r = C_p - C_p^l$, and $d(c,p) = d(c, v_l) + d(v_l, p)$ for any $c \in C_p^l$. By Equation 5,

$$\begin{aligned} &\sum_{c \in C_p^r} w(c) \cdot \big(d(c, v_l) - d(c,p)\big) \\ &> \sum_{c \in C_p^l} w(c) \cdot \big(d(c,p) - d(c, v_l)\big) = d(v_l, p) \cdot \sum_{c \in C_p^l} w(c). \end{aligned} \tag{6}$$

Since $d(c, v_l) \leq d(c,p) + d(v_l, p)$ for any $c \in C_p^r$, we have

$$d(v_l, p) \cdot \sum_{c \in C_p^r} w(c) \geq \text{LHS of (6)} \geq d(v_l, p) \cdot \sum_{c \in C_p^l} w(c),$$

which means that

$$\sum_{c \in C_p^r} w(c) > \sum_{c \in C_p^l} w(c). \tag{7}$$

Note that $d(c,p) = d(c, v_r) + d(v_r, p)$ for any $c \in C_p^r$, and $d(c, v_r) \leq d(c,p) + d(v_r, p)$ for any $c \in C_p^l$. By Eqn. 4 & 5,

$$\begin{aligned} &m(v_r) - m(p) \geq -m(p) + \sum_{c \in C_p} w(c) \cdot (a(c) - d(c, v_r)) \\ &= \sum_{c \in C_p^l} w(c) \cdot \big(d(c,p) - d(c, v_r)\big) + \sum_{c \in C_p^r} w(c) \cdot \big(d(c,p) - d(c, v_r)\big) \\ &\geq d(v_r, p) \cdot \Big(- \textstyle\sum_{c \in C_p^l} w(c) + \sum_{c \in C_p^r} w(c) \Big) \end{aligned} \tag{8}$$

By Equations 7 and 8, $m(v_r) - m(p) \geq 0$. Hence, the lemma is proved.

By Lemma 2, if the endpoints of an edge $e \in E_c$ have different merits, then the endpoint with the larger merit should be the only local MinSum location on e. But what if the merits of the endpoints are identical? The following lemma provides the answer.

Lemma 3 *Let e be an edge in E with endpoints v_l, v_r, such that $m(v_l) = m(v_r)$. Then, either all points on e have the same merit, or v_l and v_r have larger merit than any other points on e.*

Proof First of all, by Lemma 2, for any point ρ on e, it must satisfy $m(\rho) \leq m(v_l) = m(v_r)$, given that $m(v_l) = m(v_r)$. Now, assume on the contrary that there exist two points p and q on e, such that $m(v_l) = m(v_r) = m(p) \neq m(q)$. This indicates that $m(q) < m(v_l) = m(v_r) = m(p)$. Assume without loss of generality that $d(v_l, p) < d(v_l, q)$. We will prove the lemma by showing that $m(p) = m(v_l)$ cannot hold given $m(p) > m(q)$.

Let C_p be the set of clients attracted by p. We divide C_p into three subsets C_1, C_2, and C_3, such that

$$\begin{aligned} C_1 &= \{c \in C_p \mid d(c,p) = d(c,q) - d(p,q)\}, \\ C_2 &= \{c \in C_p \mid d(c,p) = d(c,q) + d(p,q)\}, \\ C_3 &= C_p - C_1 - C_2. \end{aligned}$$

It can be verified that, for any client $c \in C_3$, the shortest path from c to p must go through v_l. This indicates that,

$$d(c, v_l) = d(c, p) - d(v_l, p), \forall c \in C_3. \quad (9)$$

Given $m(p) > m(q)$ and $C_p \subseteq C$, we have

$$\sum_{c \in C_1} w(c) \cdot \big(d(c,q) - d(c,p)\big) - \sum_{c \in C_2} w(c)\big(d(c,p) - d(c,q)\big) + \sum_{c \in C_3} w(c) \cdot |d(c,q) - d(c,p)| \quad > \quad 0.$$

This leads to

$$\sum_{c \in C_1 \cup C_3} w(c) - \sum_{c \in C_2} w(c) > 0 \quad (10)$$

On the other hand, we have

$$\begin{aligned} m(v_l) - m(p) &\geq \sum_{c \in C_1 \cup C_3} w(c) \cdot \big(d(c,p) - d(c,v_l)\big) \\ &\quad - \sum_{c \in C_2} w(c) \cdot \big(d(c, v_l - d(c,p)\big) \\ &= d(v_l, p) \cdot \Big(\sum_{c \in C_1 \cup C_3} w(c) - \sum_{c \in C_2} w(c) \Big) \\ &> 0. \quad \text{(By Equation 10)} \end{aligned} \quad (11)$$

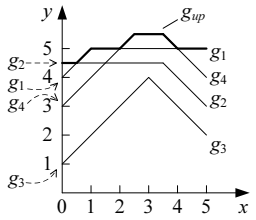
Thus, the lemma is proved.

By Lemmas 2 and 3, we can identify the local MinSum locations on any given edge e as follows. First, we compute the merits of e's endpoints based on their attraction sets. If the merits of the endpoints differ, then we return the endpoint with the larger merit as the answer. Otherwise (i.e., both endpoints of e have the same merit γ), we inspect any point p in the interior of e, and derive $m(p)$ using the attraction sets of the endpoints. If $m(p) < \gamma$, both endpoints of e are returned as the result; otherwise, we must have $m(p) = \gamma$, in which case we return the whole edge e as the answer. In summary, the local MinSum locations on e can be found by computing the merits of at most three points on e, which takes $O(n)$ time and $O(n)$ space given the attraction sets of e's endpoints.

Note that the above algorithm assumes that both endpoints of e are candidate locations. To accommodate the case when either endpoint of e is a facility, we post-process the output of our algorithm as follows. If the set S of local MinSum locations returned by our algorithm contains a facility endpoint, we set $S = \emptyset$; otherwise, we keep S intact. To understand this post-processing step, observe that the merit of any facility point is zero, since building a new facility on any point in F does not change the attractor distance of any client. Hence, if S contains a facility point, then the maximum merit of all points on e should be zero. In that case, the global MinSum location must not be on e, and hence, we can ignore the local MinSum locations found on e.

$a(c_1) = 5$
$a(c_2) = 4.5$
$a(c_3) = 6$
$a(c_4) = 5.5$

(a) Attractor Distances Without the New Facility



(b) Piecewise Linear Functions

Fig. 5 Representing Attractor Distances as Functions of the Location of the New Facility

4.3 MinMax Location Queries

Next, we present our solution for finding the local MinMax locations on any edge $e \in E_c$, i.e., the points on e where a new facility can be built to minimize the maximum WAD of all clients. Our solution is based on the following observation: For any client c, the relationship between the WAD of c and the new facility's location can be precisely captured using a piecewise linear function.

For example, consider the edge e_0 in Figure 4(a). Assume that there exist only 4 clients c_1, c_2, c_3, and c_4, as illustrated in the attraction sets in Figure 4(a). Further assume that (i) the clients' attractor distances are as shown in Figure 5(a), and (ii) all clients have a weight 1. Then, if we add a new facility on e_0 that is x ($x \in [0, 5]$) distance away from the left endpoint v_l of e_0, the WAD of c_3 can be expressed as a piecewise linear function:

$$g_3(x) = \begin{cases} x + 1, \text{ if } x \in [0, 3] \\ 7 - x, \text{ if } x \in (3, 5] \end{cases}$$

We define g_3 as the WAD *function* of c_3. Similarly, we can also derive a WAD function g_i for each of the other client c_i ($i = 1, 2, 4$). Figure 5(b) illustrates g_i ($i \in [1, 4]$).

Let g_{up} be the *upper envelope* [1] of $\{g_i\}$, i.e., $g_{up}(x) = \max_i\{g_i(x)\}$ for any $x \in [0, 5]$ (see Figure 5(b)). Then, $g_{up}(x)$ captures the maximum WAD of the clients when a new facility is built on x. Thus, if the point (on e_0) that is x distance away from v_l is a local MinMax location, then g_{up} must be minimized at x, and vice versa. As shown in Figure 5(b), g_{up} is minimized when $x \in [0, 0.5]$. Hence, the local MinMax locations on e_0 are the points p on e_0 with $d(p, v_l) \in [0, 0.5]$.

In general, to compute the local MinMax locations on an edge e, it suffices to first construct the upper envelope of all clients' WAD functions, and then identify the points at which the upper envelope is minimized. This motivates our *MinMaxLoc* algorithm (in Figure 6) for computing local MinMax locations.

Given an edge $e \in E_c$, *MinMaxLoc* first retrieves two attraction sets $\mathcal{A}(v_l)$ and $\mathcal{A}(v_r)$, where v_l (v_r) is the left (right) endpoint of e. After that, it creates a two-dimensional plane R, in which it will construct the WAD functions of

Algorithm *MinMaxLoc* (e)

1. let ℓ be the length of e, and v_l (v_r) be the left (right) endpoint of e
2. construct an empty two-dimensional plane R
3. let C_- be the set of clients that appear in neither $\mathcal{A}(v_l)$ nor $\mathcal{A}(v_r)$
4. find the client $c_0 \in C_-$ with the largest WAD
5. construct the WAD function of c_0, i.e., draw in R a line segment from point with coordinate $\big(0, \widehat{a}(c_0)\big)$ to point with coordinate $\big(\ell, \widehat{a}(c_0)\big)$
6. let C_Δ be the set of clients that appear in both $\mathcal{A}(v_l)$ and $\mathcal{A}(v_r)$
7. for each client $c \in C - C_\Delta - C_-$
8. if c appears in $\mathcal{A}(v_l)$
9. $x_1 = 0, \quad y_1 = w(c) \cdot d(c, v_l)$
10. $x_3 = \ell, \quad x_2 = \min\{\ell, a(c) - d(c, v_l)\}$
11. $y_2 = y_3 = w(c) \cdot \big(x_2 + d(c, v_l)\big)$
12. else /*if c does not appear in $\mathcal{A}(v_l)$, but appears in $\mathcal{A}(v_r)$*/
13. $x_1 = \ell, \quad y_1 = w(c) \cdot d(c, v_r)$
14. $x_3 = 0, \quad x_2 = \max\{0, \ell - a(c) + d(c, v_r)\}$
15. $y_2 = y_3 = w(c) \cdot \big(\ell - x_2 + d(c, v_r)\big)$
16. construct the WAD function of c, i.e., draw in R two line segments, from (x_1, y_1) to (x_2, y_2), then to (x_3, y_3)
17. for each client $c \in C_\Delta$ /*c appears in both $\mathcal{A}(v_l)$ and $\mathcal{A}(v_r)$*/
18. $x_1 = 0, \quad y_1 = w(c) \cdot d(c, v_l)$
19. $\beta = \frac{1}{2}\ell - \frac{1}{2}d(c, v_l) + \frac{1}{2}d(c, v_r)$
20. $x_2 = \min\{\beta, a(c) - d(c, v_l)\}, \quad y_2 = w(c) \cdot \big(x_2 + d(c, v_l)\big)$
21. $x_3 = \max\{\beta, \ell - a(c) + d(c, v_r)\}, \quad y_3 = y_2$
22. $x_4 = \ell, \quad y_4 = w(c) \cdot d(c, v_r)$
23. construct the WAD function of c, i.e., draw in R three line segments, from (x_1, y_1) to (x_2, y_2), then to (x_3, y_3), then to (x_4, y_4)
24. compute the upper envelope g_{up} of the WAD functions in R
25. identify and return the points on which g_{up} is minimized

Fig. 6 The *MinMaxLoc* Algorithm

some clients. Specifically, *MinMaxLoc* first identifies the set C_- of clients that appear in neither $\mathcal{A}(v_l)$ nor $\mathcal{A}(v_l)$. By Lemma 1, for any client $c \in C_-$, the attractor distance of c is not affected by a new facility built on e. Hence, the WAD function of c can be represented by a horizontal line segment in R. Observe that, only one of those segments may affect the upper envelope g_{up}, i.e., the segment corresponding to the client c^* with the largest WAD in C_-. Therefore, given C_-, *MinMaxLoc* only constructs the WAD function of c^*, ignoring all the other clients in C_-.

Next, *MinMaxLoc* examines each client $c \in C - C_-$, and derive the WAD function of c based on $\mathcal{A}(v_l)$ and $\mathcal{A}(v_r)$. In particular, each WAD function is represented using at most three line segments in R. Finally, *MinMaxLoc* computes the upper envelope g_{up} of the WAD functions in R, and then identifies and returns the points at which g_{up} is minimized.

MinMaxLoc can be implemented in $O(n \log n)$ time and $O(n)$ space. Specifically, given the attractor distances of the clients in C, we can identify the client c^* with $O(n)$ time and space. After that, it takes only $O(n)$ time and space to construct the WAD functions of clients, since each function is represented with $O(1)$ line segments. As there exist $O(n)$ segments in R, the upper envelope g_{up} should contain $O(n)$ linear pieces, and can be computed in $O(n \log n)$ time and $O(n)$ space [13]. Finally, by scanning the $O(n)$ linear pieces of g_{up}, we can compute the local MinMax locations on e in $O(n)$ time and space.

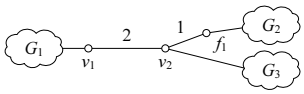


Fig. 7 Example of Lemma 4

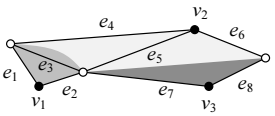


Fig. 8 Example of *GPart*

In addition, *MinMaxLoc* can also be extended to handle the case when either endpoint of e is a facility in F. In particular, if the left endpoint v_l of e is a facility, then *MinMaxLoc* excludes v_l when it computes the upper envelope g_{up} of the WAD functions. That is, the domain of g_{up} is defined as $(0, \ell]$ instead of $[0, \ell]$. The case when the right endpoint of e is a facility can be addressed similarly.

5 Computing Attraction Sets and Attractor Distances

Our algorithms in Section 4 require as input (i) the attractor distances of all clients in C, and (ii) the attraction sets of the endpoints of the given edge $e \in E_c$. The attractor distances can be easily computed using the algorithm by Erwig and Hagen [7]. Specifically, Erwig and Hagen's algorithm takes as input a road network G and a set F of facilities. With $O(n \log n)$ time and $O(n)$ space, the algorithm can identify the distance from *each vertex v in G to its nearest facility in F*. In the following, we will investigate how to compute the attraction sets of the vertices in G, given the attractor distances derived from Erwig and Hagen's algorithm.

5.1 The Blossom Algorithm

By definition, a client c appears in the attraction set of a vertex v, if and only if $d(c, v)$ is no more than the attractor distance $a(c)$ of c. Therefore, given the attractor distances of all clients, we can compute the attraction sets of all vertices in G in a batch as follows. First, we set the attraction set of every vertex in G to $\emptyset$. After that, for each client $c \in C$, we apply Dijkstra's algorithm [5] to traverse the vertices in G in ascending order of their distances to c. For each vertex v encountered, we check whether $d(c, v) \leq a(c)$. If $d(c, v) \leq a(c)$, c is inserted into the attraction set of v. Otherwise, $d(c, v') > a(c)$ must hold for any unvisited vertex v', i.e., none of the unvisited vertices can attract c. In that case, we terminate the traversal and proceed to the next client. Once all clients are processed, we obtain the attraction sets of all vertices in G. We refer to the above algorithm as *Blossom*, as illustrated in Figure 9.

Blossom has an $O(n^2 \log n)$ time complexity, since it invokes Dijkstra's algorithm once for each client, and each execution of Dijkstra's algorithm takes $O(n \log n)$ time in the

Algorithm *Blossom* (G)

1. initialize the attraction set of each vertex v in G as $\emptyset$
2. for each client c
3. employ Dijkstra's algorithm to traverse the vertices in G in ascending order of their distances to c
4. for each vertex v traversed
5. if $d(c, v) \leq a(c)$
6. add an entry $\langle c, d(c, v)\rangle$ to the attraction set of v
7. else goto Line 2
8. return

Fig. 9 The *Blossom* Algorithm

worst case [5]. *Blossom* requires $O(n^2)$ space, as it materializes the attraction set of each vertex in G, and each attraction set contains the information of $O(n)$ clients. Such space consumption is prohibitive when n is large. To remedy this deficiency, Section 5.2 proposes an alternative solution that requires only $O(n)$ space.

5.2 The OTF Algorithm

The enormous space requirement of *Blossom* is caused by the massive materialization of attraction sets. A natural idea to reduce the space overhead is to avoid storing the attraction sets, and derive them only when needed. That is, whenever we need to compute the local optimal locations on an edge $e \in E_c$, we compute the attraction sets of e's endpoints *on the fly*, and then discard the attraction sets once the local optimal locations are found. But the question is, given a vertex v in G, how do we construct the attraction set $\mathcal{A}(v)$ of v? A straightforward solution is to apply Dijkstra's algorithm to scan through all vertices in G in ascending order of their distances to v. In particular, for each client c encountered during the scan, we examine the distance $d(v, c)$ from c to v. If $d(v, c) < a(c)$, we add c into $\mathcal{A}(v)$; otherwise, c is ignored. Apparently, this solution incurs significant computation overhead, as it requires traversing a large number of vertices. Is it possible to compute $\mathcal{A}(v)$ without an exhaustive search of the vertices in G? The following lemma provides us some hints.

Lemma 4 *Given two vertices v and v' in G, such that $d(v, v')$ is larger than the distance from v' to its nearest facility f'. Then, $\forall c \in \mathcal{A}(v)$, the shortest path from v to c must not go through v'.*

Proof Assume on the contrary that the shortest path from v to c goes through v'. Then, we have

$$d(v, c) = d(v, v') + d(v', c) > d(v', f') + d(v', c) \geq d(f', c).$$

This contradicts our assumption that c is attracted by v.

Algorithm *OTF* (v)

1. initialize the attraction set $\mathcal{A}(v)$ of v as $\emptyset$
2. employ Dijkstra's algorithm to traverse the vertices in G in ascending order of their distances to v
3. for each vertex v' examined
4. let λ be distance from v' to its closest facility
5. if $d(v, v') \leq \lambda$ and v' is a client point
6. insert an entry $\langle v',\ dist(v', v)\rangle$ into $\mathcal{A}(v)$
7. if $d(v, v') > \lambda$
8. ignore all edges adjacent to v', i.e., regard them as deleted
9. if none of the unvisited vertices can be reached from v
10. return $\mathcal{A}(v)$
11. return $\mathcal{A}(v)$

Fig. 10 The *OTF* Algorithm

Consider for example the road network in Figure 7, which contains three subgraph G_1, G_2, and G_3 that are connected by a facility f_1 and two vertices v_1 and v_2. In addition, $d(v_1, v_2) = 2$, $d(v_2, f_1) = 1$, and f_1 is the facility closest to v_2. Since $d(v_1, v_2) > d(v_2, f_1)$, by Lemma 4, v_2 must not be on the shortest path from v_1 to any client attracted by v_1. This means that no client in G_2 or G_3 can be attracted by v_1, because all paths from v_1 to G_2 or G_3 go through v_2. Therefore, if we are to compute $\mathcal{A}(v_1)$, it suffices to examine only the clients in G_1.

Based on Lemma 4, we propose the *OTF (On-The-Fly)* algorithm (in Figure 10) for computing the attraction set of a vertex v in G. Given v, *OTF* first sets $\mathcal{A}(v) = \emptyset$, and then applies Dijkstra's algorithm to visit the vertices in G in ascending order of their distances to v. For each vertex v' visited, *OTF* retrieves the distance λ from v' to its closest facility (recall that λ is computed using Erwig and Hagen's algorithm [7]). If $d(v, v') \leq \lambda$ and v' is a client, then *OTF* adds v' into $\mathcal{A}(v)$. On the other hand, if $d(v, v') > \lambda$, then *OTF* ignores all edges adjacent to v' when it traverses the remaining vertices in G. This does not affect the correctness of *OTF*, since, by Lemma 4, deleting v' from G does not change the shortest path from v to any client attracted by v. After v is processed, *OTF* checks whether any of the unvisited vertices in G is still connected to v. If none of those vertices is connected to v, *OTF* terminates by returning $\mathcal{A}(v)$; otherwise, *OTF* proceeds to the unvisited vertex that is closest to v.

It is not hard to verify that *OTF* runs in $O(n \log n)$ time and $O(n)$ space. Therefore, if we employ *OTF* to compute the local optimal locations on every edge in G, then the total time required for deriving the attraction sets would be $O(n^2 \log n)$, and the total space needed is $O(n)$ (as *OTF* does not materialize any attraction sets). In contrast, computing the attraction sets with *Blossom* incurs $O(n^2 \log n)$ time and $O(n^2)$ space overhead. Hence, *OTF* is more favorable than *Blossom* in terms of asymptotic performance.

6 Pruning of Road Segments

Given the algorithms in Sections 4 and 5, we may answer any OL query by first enumerating the local optima on each edge in E_c, and then deriving the global optimal solutions based on the local optima. This approach, however, incurs a significant overhead when E_c contains a large number of edges. To address this issue, in this section we propose a *fine-grained partitioning (FGP)* technique to avoid the exhaustive search on the edges in E_c.

6.1 Algorithm Overview

At a high level, FGP works in four steps as follows. First, we divide G into m edge-disjoint subgraphs $G_1, G_2, \ldots, G_m$, where m is an algorithm-specific parameter. Second, for each subgraph G_i ($i \in [1, m]$), we derive a *potential client set* C_i of G_i, i.e., a superset of all clients that can be attracted by a new facility built on any edge in G_i.

As a third step, we inspect each potential client set C_i ($i \in [1, m]$), based on which we derive an upper-bound of the *benefit* of any candidate location p in G_i. Specifically, for competitive location queries, the benefit of p is defined as the total weight of the clients attracted by p; for MinSum (MinMax) location queries, the benefit of p is quantified as the reduction in the total (maximum) WAD of all clients, when a new facility is built on p.

Finally, we examine the subgraphs $G_1, G_2, \ldots, G_m$ in descending order of their benefit upper-bounds. For each subgraph G_i, we apply the algorithms in Sections 4 and 5 to identify the local optimal locations in G_i. After processing G_i, we inspect the set S of best local optima we have found so far. If the benefits of those locations are larger than the benefit upper-bounds of all unvisited subgraphs, we terminate the search and return S as the final results; otherwise, we move on to the next subgraph.

The efficacy of the above framework rely on three issues, namely, (i) how the subgraphs of G are generated, (ii) how the potential client set C_i of each subgraph G_i is derived, and (iii) how the benefit upper bound of G_i is computed. In the following, we will first clarify how FGP derives benefit upper-bounds, deferring the solutions to the other two issues to Section 6.2. We begin with the following lemma.

Lemma 5 *Let G_i be a subgraph of G, C_i be the potential client set of G_i, and p be a candidate location in G_i. Then, for any competitive location query, the benefit of p is at most $\sum_{c \in C_i} w(c)$. For any MinSum location query, the benefit of p is at most $\sum_{c \in C_i} \widehat{a}(c)$. For any MinMax location query, the benefit of p is at most $\max_{c \in C} \widehat{a}(c) - \max_{c \in C - C_i} \widehat{a}(c)$.*

Algorithm *GPart* (G, θ)

1. construct a set V' that contains all the endpoints of the edges that appear in both G and E_c
2. randomly sample a set V_Δ of vertices from V' with a sampling rate θ
3. create $|V_\Delta|$ empty subgraphs, and assign each vertex in V_Δ as the "center" of a distinct subgraph
4. feed G and V_Δ as input to Erwig and Hagen's algorithm [7] to compute, for each vertex v in G, (i) the vertex $v' \in V_\Delta$ that is closest to v, as well as (ii) the distance $d(v, v')$ from v to v'
5. insert each edge in G to the subgraph whose center is the closest to either endpoint of the edge
6. return all subgraphs

Fig. 11 The *GPart* Algorithm

Proof The lemma follows from the facts that (i) C_i contains all clients that can be attracted by p, and (ii) for any client in C_i, its WAD is at least zero after a new facility is built on p.

The benefit upper-bounds in Lemma 5 require knowledge of all clients' attractor distances, which, as mentioned in Section 5, can be computed in $O(n \log n)$ time and $O(n)$ space using Erwig and Hagen's algorithm [7]. We can further sort the attractor distances in a descending order with $O(n \log n)$ time and $O(n)$ space. Observe that, given the sorted attractor distances and the potential client set C_i of a subgraph G_i, the benefit upper-bound of G_i (for any OL query) can be computed efficiently in $O(|C_i|)$ time and $O(n)$ space.

6.2 Graph Partitioning

We are now ready to discuss how FGP generates the subgraphs from G and computes the potential client set of each subgraph. In particular, FGP generates subgraphs from G by applying an algorithm called *GPart* (as illustrated Figure 11), which takes as input G and a user defined parameter $\theta \in (0, 1]$. *GPart* first identifies the set V of vertices in G that are adjacent to some edges in E_c. As a second step, *GPart* computes a random sample set V_Δ of the vertices in V with a sampling rate θ, after which it splits G into subgraphs based on V_Δ.

Specifically, *GPart* first constructs $|V_\Delta|$ empty subgraphs, and assigns each vertex in V_Δ as the "center" of a distinct subgraphs. After that, for each vertex v in G, *GPart* identifies the vertex $v' \in V_\Delta$ that is the closest to v, and computes $d(v, v')$. This step can be done by applying Erwig and Hagen's algorithm [7], with G and V_Δ as the input. Next, for each edge e in G, *GPart* checks the two endpoints v_l and v_r of e, and inserts e into the subgraph whose center v' minimizes $\min\{d(v', v_l), d(v', v_r)\}$. After all edges in G are processed, *GPart* terminates by returning all subgraphs constructed. For example, if G equals the graph in Figure 8 and

Algorithm *P-OTF* (G, G_i)

1. let V_i' be the set of endpoints of the edges in $G_i \cap E_c$
2. insert a new vertex v_0 in G
3. connect v_0 to each vertex in V_i' via an edge with a length 0
4. compute the attraction set of v_0 in G using the *OTF* algorithm
5. return the set of clients in the attraction set of v_0

Fig. 12 The *P-OTF* Algorithm

$V_\Delta = \{v_1, v_2, v_3\}$, then *GPart* would construct three subgraphs $\{e_1, e_2, e_3\}$, $\{e_4, e_5, e_6\}$, and $\{e_7, e_8\}$, whose centers are v_1, v_2, and v_3, respectively. In summary, *GPart* ensures that the edges in the same subgraph form a cluster around the subgraph center. As such, the edges belonging to the same subgraph tend to be close to each other. This helps tighten the benefit upper-bounds of the subgraph because, intuitively, points in proximity to each other have similar benefits. Regarding asymptotic performance, it can be verified that *GPart* runs in $O(n \log n)$ time and $O(n)$ space.

Given a subgraph G_i obtained from *GPart*, our next step is to derive the potential client set for each G_i. Let E_{G_i} be the set of edges in G_i that also appear in E_c, and V_{G_i} be the set of endpoints of the edges in E_{G_i}. By Lemma 1, for any candidate location p on an edge e in G_i, the set of clients attracted by p is always a subset of the clients in the attraction sets of e's endpoints. Therefore, the potential client set of G_i can be formulated as the set of all clients that appear in the attraction sets of the vertices in V_{G_i}. In turn, the attraction sets of the vertices in V_{G_i} can be derived by applying either the *Blossom* algorithm or the *OTF* algorithm in Section 5. In particular, if *Blossom* is adopted, then we feed the graph G as the input to *Blossom*[1]. In return, we obtain the attraction sets of all vertices in G, based on which we can compute the potential clients set of *all* subgraphs in G. In addition, the attraction sets can be reused when we need to compute the local optimal locations on any edge in G.

On the other hand, if *OTF* is adopted, then we feed each vertex in V_{G_i} to *OTF* to compute its attraction set, after which we collect all clients that appear in at least one of the attraction sets. The drawback of this approach is that it requires multiple executions of *OTF*, which leads to inferior time efficiency. To remedy this drawback, we propose the *P-OTF* algorithm (in Figure 12) for computing the potential client set of a subgraph G_i. Given the graph G, *P-OTF* first creates a new vertex v_0 in G, and then constructs an edge between v_0 and each vertex in V_{G_i}, such that the edge has a length 0. After that, *P-OTF* invokes *OTF once* to compute the attraction set of v_0 in G. Observe that, if a client c is attracted by v_0, then there must exist a vertex in V_{G_i} that attracts c, and vice versa. Hence, the potential client set of G_i should be equal to the set of clients in the attraction set $\mathcal{A}(v_0)$ of v_0. Therefore, once $\mathcal{A}(v_0)$ is computed, *P-OTF* terminates by returning all clients in $\mathcal{A}(v_0)$. In summary, *P-OTF* computes the potential client set of G_i by invoking *OTF* only once, which incurs $O(n \log n)$ time and $O(n)$ space overhead.

Before closing this section, we discuss how we set the input parameter θ of *GPart*. In general, a larger θ results in smaller subgraphs, which in turn leads to tighter benefit upper-bounds. Nevertheless, the increase in θ would also lead to a larger number of subgraphs, which entails a higher computation cost, as we need to derive the potential client set for each subgraph. Ideally, we should set θ to an appropriate value that strikes a good balance between the tightness of the benefit upper-bounds and the cost of deriving the bounds. We observe that, when the potential client sets of the subgraphs are computed using *Blossom*, θ should be set to 1. This is because, *Blossom* derives potential client sets by computing the attraction sets of *all* vertices, regardless of the value of θ. As a consequence, the computation cost of the benefit upper-bounds is independent of θ. Hence, we can set $\theta = 1$ to obtain the tightest benefit upper-bounds without sacrificing time efficiency. On the the hand, if *P-OTF* is adopted, then the overhead of computing potential client sets increases with the number of subgraphs. To ensure that this computation overhead does not affect the overall performance, θ should be set to a small value. We suggest setting $\theta = 1‰$ across the board.

7 Recomputation of Local Optimal Locations after Updates

In this section, we present solutions for efficiently monitor the local OLs given continuous updates in the locations of facilities and clients. This problem setting can be illustrated with following example.

Example 5 John owns a set of trucks for food service in San Francisco, and he would like to position his trucks to attract the largest number of clients against competition from other food-service trucks. Given that the locations of clients and other competing trucks may change as time evolves, John would like to continuously monitor the optimal locations for positioning his trucks. ■

The updates of facilities and/or clients on a road-network are natural and common operations. For users who would like to track optimal location(s) in the face of updates over a time period, our dynamic monitoring techniques are essential. Note that tracking optimal location(s) over a period of time, rather than finding an optimal location in a given time-instance and building a facility there right away, often is a natural choice for many business operations. For example, a corporation would like to monitor the optimal location for

[1] Note that we cannot apply *Blossom* on G_i directly, since a candidate location in G_i may attract a client outside G_i.

their business to build a new facility in a chosen time period, before making the final decision. This gives them the opportunity to carry out market analysis, learn how other facilities may update their locations, and more importantly, how clients may move on the road network over a period of time.

Furthermore, our dynamic monitoring techniques can be used to answer *continuous optimal location queries* efficiently, where a service provider needs to answer a sequence of optimal location queries from different users, and very naturally, updates may take place in-between any two OLQs. Instead of always answering an OLQ from scratch, it is obviously desirable to answer them in an incremental and monitoring fashion.

In what follows, we will first present an overview of our solutions for handling updates (Section 7.1), and then discuss detailed algorithms for the three variants of optimal location queries (Sections 7.2-7.4).

7.1 Overview

We consider four types of updates to the locations of clients and facilities:

1. Insertion of a client c (denoted as $AddC(c)$);
2. Deletion of a client c (denoted as $DelC(c)$);
3. Insertion of a facility f (denoted as $AddF(f)$);
4. Deletion of a facility f (denoted as $DelF(f)$).

A straightforward method for handle such updates is to adopt our algorithms in Section 3-6, i.e., we recompute the OLs from scratch after each update. Intuitively, this method is highly inefficient as it performs a large amount of redundant computation in the identification of OLs. To address this issue, we extend our unified framework in Section 3 and enables it to *incrementally* process updates.

First, for each edge $e \in E_c$, we maintain the set I_0 of local OLs on e, as well as the benefit m_0s of those OLs. For the competitive location query, the benefit m_0 is the total weights of clients attracted by a facility built on the OLs. For the MinSum location query, m_0 is the merit of the OLs. For the MinMax location query, m_0 is the maximum WAD of all clients. Given an update of clients or facilities, we process the update in four steps as follows:

1. **Step 1:** We compute a set V_c of clients whose attractor distances are affected by the update.
2. **Step 2:** For each client $c \in V_c$, we identify its previous attractor distance $a^0(c)$ and new attractor distance $a'(c)$, and we construct two sets $\mathcal{U}_c^-$ and $\mathcal{U}_c^+$, where

$$\mathcal{U}_c^- = \{\langle v, d(c,v)\rangle | d(c,v) < a^0(c)\},$$
$$\mathcal{U}_c^+ = \{\langle v, d(c,v)\rangle | d(c,v) < a'(c)\}.$$

3. **Step 3:** We update I_0 and m_0 for each edge e, based on the sets V_c, $a^0(c)$, $a'(c)$, $\mathcal{U}_c^-$, and $\mathcal{U}_c^+$ associated with each client $c \in V_c$.
4. **Step 4:** We derive and return the global OLs when none of the unvisited $e \in E_c$ can provide a better solution than the best optima found so far.

To explain, recall that the local OLs on an edge e are decided by (i) the attraction sets for the endpoints of e and (ii) the attractor distances of the clients. Therefore, if we are to derive the new I and m for each edge, we can first identify the changes in the aforementioned attraction sets and attractor distances. Steps 1 and 2 serve this purpose since (i) for any client c, the change in the attractor distance of c can be derived from $a^0(c)$ and $a'(c)$, and (ii) the change in the attraction sets of e's endpoints can be derived given $\mathcal{U}_c^-$ and $\mathcal{U}_c^+$ for each c. In the following, we will first discuss how Steps 1 and 2 can be implemented for the four types of updates, namely, $AddC(c)$, $DelC(c)$, $AddF(f)$, and $DelF(f)$.

AddC(c)* and *DelC(c): Given a client c to be inserted or deleted, we can easily perform Step 1 by setting $V_c = \{c\}$. For Step 2, we employ the Dijkstra's algorithm to traverse the vertices in G in ascending order of their distances to c, until we reach the facility f nearest to c. Then, we set $a_0(c) = 0$ and $a'(c) = d(c,f)$ if f is just inserted; otherwise (i.e., f is just deleted), we set $a_0(c) = d(c,f')$ and $a'(c) = d(c,f)$. Finally, for each vertex v visited by Dijkstra's algorithm, we insert an entry $\langle v, d(c,v)\rangle$ into $\mathcal{U}_c^+$.

AddF(f)* and *DelF(f): When a facility f is inserted or deleted, the set V_c of clients in Step 1 is exactly the set $\mathcal{A}(f)$, i.e., the attraction set of f. To compute $\mathcal{A}(f)$, we can use either the *Blossom* algorithm in Section 5.1 or the *OTF* algorithm in Section 5.2. After that, we can perform Step 2 as follows. For each client $c \in \mathcal{A}(f)$, we employ Dijkstra's algorithm to traverse the vertices in G in ascending order of their distances to c, until we reach a facility $f' \neq f$. Then, we set $a_0(c) = d(c,f')$ and $a'(c) = d(c,f)$. Finally, for each vertex v visited Dijkstra's algorithm, we insert an entry $\langle v, d(c,v)\rangle$ into $\mathcal{U}_c^-$ if $d(c,v) \leq d(c,f)$; otherwise, we insert an entry $\langle v, d(c,v)\rangle$ into $\mathcal{U}_c^+$.

Next, we clarify how Step 3 can be implemented. In our solution, for each $c \in V_c$, we update the local OLs for each edge using the change in attractor distances and attraction sets caused by c, which can be derived from $a^0(c), a'(c), \mathcal{U}_c^-$ and $\mathcal{U}_c^+$. Note that not all edges' local OLs can be affected by a client c. In other words, when we update local OLs for each edge with the effect of c, we can initially identify a set of edges whose I and m will remain unchanged. Therefore, for each $c \in V_c$, we first prune this set of edges and then update the local OLs for remaining edges. In the following, we will clarity how we can perform the pruning of edges and

Algorithm *CompLoc_ClientUpdate* (c)

1. initialize an empty set of edges E_c
2. for each edge $e(v_l, v_r) \in E$
3. if $\langle v_l, d(c, v_l)\rangle \in \mathcal{U}_c^- \cup \mathcal{U}_c^+$ or $\langle v_r, d(c, v_r)\rangle \in \mathcal{U}_c^- \cup \mathcal{U}_c^+$
4. insert e into E_c
5. for each $e(v_l, v_r) \in E_c$
6. initialize two empty sets of intervals I^-, I^+
7. if $\langle v_l, d(c, v_l)\rangle \in \mathcal{U}_c^-$ and $\langle v_r, d(c, v_r)\rangle \notin \mathcal{U}_c^-$
8. add a line segment $[0, a^0(c) - d(c, v_l)]$ into I^-
9. if $\langle v_l, d(c, v_l)\rangle \notin \mathcal{U}_c^-$ and $\langle v_r, d(c, v_r)\rangle \in \mathcal{U}_c^-$
10. add a line segment $[\ell - a^0(c) + d(c, v_r), \ell]$ into I^-
11. if both $\langle v_l, d(c, v_l)\rangle \in \mathcal{U}_c^-$ and $\langle v_r, d(c, v_r)\rangle \in \mathcal{U}_c^-$
12. if $\ell \leq 2 \cdot a^0(c) - d(c, v_l) - d(c, v_r)$
13. add a line segment $[0, \ell]$ into I^-
14. else
15. add two line segments $[0, a^0(c) - d(c, v_l)]$ and $[\ell - a^0(c) + d(c, v_r), \ell]$ into I^-
16. compute I^+ similarly to line 7-15 but use $\mathcal{U}_c^+$ instead of $\mathcal{U}_c^-$, $a'(c)$ instead of $a^0(c)$
17. if $a^0(c) < a'(c)$
18. set $flag = ADD$, $I' = I^+ - I^-$
19. else
20. set $flag = DEL$, $I' = I^- - I^+$
21. if I' is empty
22. continue to next e
23. if $flag$ is ADD
24. $I = I_0 \cap I'$
25. if I is empty
26. recompute I and m using the *CompLoc* Algorithm in Figure 3
27. else
28. $m = m_0 + w(c)$
29. if $flag$ is DEL
30. if $I' = [0, \ell]$
31. $m = m_0 - w(c)$, $I = I_0$
32. else
33. $I = I_0 - I'$
34. if I is empty
35. recompute I and m using the *CompLoc* Algorithm in Figure 3
36. else
37. $m = m_0$
38. keep I, m for e instead of I_0, m_0
39. return I, m (I_0, m_0 if unchanged) for each edge e

Fig. 13 The *CompLoc_ClientUpdate* Algorithm

the update of the local OLs, for each of the three variants of OL queries.

7.2 Competitive Location Queries

Figure 13 shows the algorithm for updating local OLs for a competitive location query, given a client $c \in V_c$. The algorithm first prunes the edges whose I and m will not change (Lines 1-4 in Figure 13. Specifically, if an edge's endpoints cannot attract c either before the update or after the update, its local OLs will not change. The reason is that, when we compute local OLs on $e(v_l, v_r)$ (see Section 4.1), we use $\mathcal{A}(v_l)$, $\mathcal{A}(v_r)$ and $a(c)$s only if c appears in $\mathcal{A}(v_l)$ or $\mathcal{A}(v_r)$.

Algorithm *MinSumLoc_ClientUpdate* (c)

1. construct a set of vertices $S = \{v | \langle v, d(c, v)\rangle \in \mathcal{U}_c^- \cup \mathcal{U}_c^+\}$
2. for each $v \in S$
3. if $\langle v, d(c, v)\rangle \in \mathcal{U}_c^-$ and $\langle v, d(c, v)\rangle \notin \mathcal{U}_c^+$
4. set $m(v) = m(v) - w(c) \cdot (a^0(c) - d(v, c))$
5. if $\langle v_l, d(c, v)\rangle \notin \mathcal{U}_c^-$ and $\langle v, d(c, v)\rangle \in \mathcal{U}_c^+$
6. set $m(v) = m(v) + w(c) \cdot (a'(c) - d(v, c))$
7. if both $\langle v_l, d(c, v)\rangle \in \mathcal{U}_c^-$ and $\langle v, d(c, v)\rangle \in \mathcal{U}_c^+$
8. set $m(v) = m(v) + w(c) \cdot (a'(c) - a^0(c))$
9. return $m(v_l), m(v_r)$ for each edge $e(v_l, v_r)$

Fig. 14 The *MinSumLoc_ClientUpdate* Algorithm

As a next step, we update the local OLs for each of the remaining edges (E_c). First, we construct the set I^- of points on e that attracts c before the update, and the set I^+ of points on e that attracts c after the update (lines 6 - 16), in a similar fashion as what we do in the *CompLoc* Algorithm (Figure 3) for each client. There are three possible cases:

1. $I^- = I^+$, i.e., the set of points that attracts c remains unchanged. In this case, we keep I_0 and m_0 as new local OLs and their benefits of e.
2. $I^- \subset I^+$, i.e., there is a new set of points that can attract c. Let $I' = I^+ - I^-$. In this case, we compute $I_0 \cap I'$. If $I_0 \cap I' = \emptyset$, then we recompute the new local OLs. If $I_0 \cap I' \neq \emptyset$, then $I_0 \cap I'$ must be the the new local OLs, in which case we update m accordingly.
3. $I^+ \subset I^-$, i.e., there is a set of points that can attract c before the update but cannot attract c any more after the update. Let $I^* = I^- - I^+$. In this case, if I^* fully covers e, then the local OLs remain the same, and $m = m_0 - w(c)$. Otherwise, we compute $I_0 - I^*$. If $I_0 - I^*$ is not empty, the OLs will be $I_0 - I'$ (lines 30 and 31). On the other hand, if $I_0 - I^*$ is empty, then we recompute the local optima using the *CompLoc* Algorithm in Figure 3.

Note that one of the three cases must occur since we have $I^- \subset I^+$ whenever $a^0(c) \leq a'(c)$, and $I^+ \subset I^-$ otherwise.

Given the *CompLoc_ClientUpdate* algorithm, we can implement Step 3 in Section 7.1 as follows. For $AddC(c)$ and $DelC(c)$, we have $V_c = \{c\}$, and hence, we only need to invoke *CompLoc_ClientUpdate* once, with c as the input. For $AddF(f)$ and $DelF(f)$, we need to apply *CompLoc_ClientUpdate* once for each client in $V_c = \{c | \langle c, d(c, v)\rangle \in \mathcal{A}(f)\}$. Such multiple execution of the algorithm may lead to repeated computation of the local OLs on an edge e. To avoid this unnecessary overhead, when handling $AddF(f)$ and $DelF(f)$, we will first identify the edges whose local OLs need to be updated, and we compute the local OLs for each of those edge in a batch, without any redundant computation.

Algorithm *MinMaxLoc_ClientUpdate* (c)

1. initialize an empty set of edges E_c
2. for each edge $e \in E$
3. if $w(c) \cdot max\{a^0(c), a'(c)\} \geq m_0$
4. insert e into E_c
5. for each $e \in E_c$
6. if $w(c) \cdot a^0(c) \geq m_o$
7. use $a^0(c)$ to construct $g_c(x)$ on e
8. if $max\{g_c(x)\} < m_0$
9. $m = m_0$, $I = I_0$
10. else
11. recompute I and m using the *MinMaxLoc* Algorithm
12. if $w(c) \cdot a'(c) \geq m_o$
13. use $a'(c)$ to construct $g_c(x)$ on e
14. construct $g'_{up}(x) = max\{g_c(x), g_0(x) = m_0(0 \leq x \leq \ell)\}$
15. construct a set of intervals $I' = \text{argmin}_x\, g'_{up}(x)$
16. if $min\{g'_{up}(x)\} = m_0$ and $I' \cap I_0 \neq \emptyset$
17. set $m = m_0$, $I = I' \cap I_0$
18. else
19. recompute I and m using the *MinMaxLoc* Algorithm
20. keep I, m for e instead of I_0, m_0
21. return $I, m(I_0, m_0$ if unchanged) for each edge e

Fig. 15 The *MinMaxLoc_ClientUpdate* Algorithm

7.3 MinSum Location Queries

Recall that, to compute the local OLs for a MinSum location query, we only need the merit value of an edge's two endpoints, $m(v_l)$ and $m(v_r)$. Hence, we only need to update the merit value for each vertex v, which can attract those clients affected by the update (i.e. V_c). There are three conditions in which a vertex's merit value has to be updated. In the first case, v can attract c before the update but cannot attract c after the update. In this case we should subtract $w(c) \cdot (a^0(c) - d(v, c))$ away from $m(v)$. Secondly, v can now attract c thus we should add $w(c) \cdot (a'(c) - d(v, c))$ to $m(v)$. The last condition is that v can attract c both before and after the update but $a(c)$ has changed. In this case we should update $m(v)$ with the difference between $a^0(c)$ and $a'(c)$. The details of the *MinSumLoc_ClientUpdate* algorithm are shown in the Figure 14. To achieve step 3 during an update for a MinSum location query, we execute the *MinSumLoc_ClientUpdate* Algorithm for each $c \in V_c$.

7.4 MinMax Location Queries

Recall that the local MinMax locations on an edge e are the points x on e at which $g_{up}(x)$ is minimized (see Section 4.3). Therefore, for any edge e, if m_0 is larger than both $\widehat{a}^0(c)$ and $\widehat{a}'(c)$, then e would not be affected by the change in the attractor distance of c. In that case, we can omit e when processing the update.

On the other hand, if m_0 is smaller than either $\widehat{a}^0(c)$ or $\widehat{a}'(c)$, then we first check if the previous $g_c(x)$ can affect e's $g_{up}(x)$. If it does, we should evaluate how removing $g_c(x)$ can affect e's I and m. Secondly, we check if the new $g_c(x)$ can affect e's $g_{up}(x)$. If it does, we should evaluate how adding $g_c(x)$ can affect e's I and m. Here the previous $g_c(x)$ can be easily derived from $a^0(c)$ and $\mathcal{U}_c^-$ while the new $g_c(x)$ can be derived from $a'(c)$ and $\mathcal{U}_c^+$ (see Section 4.3).

To evaluate how removing $g_c(x)$ from e can affect I and m, if the maximum value of $g_c(x)$ is less than m_0, the local MinMax locations remain the same; otherwise, we recompute them using the *MinMaxLoc* algorithm in Figure 6. To evaluate how adding $g_c(x)$ to e may affect I and m, we compute the envelope of $g(x) = m_0$ and $g_c(x)$ as $g'_{up}(x)$. After that, if there exist points in I_0 where $g'_{up}(x)$ is minimized with value m_0, we return those points with local optima m_0. Otherwise, we recompute the local MinMax locations using the *MinMaxLoc* algorithm. The details of the *MinMaxLoc_ClientUpdate* algorithm are shown in Figure 15.

To implement Step 3 during an update for a MinMax location query, we execute the *MinMaxLoc_ClientUpdate* Algorithm for each $c \in V_c$. If the type of update is $AddF(f)$ or $DelF(f)$, we can delay the recomputation operation to the end of update in order to avoid redundant computations, as we discussed in Section 7.2.

8 Updates with FGP

In this section, we discuss how the FGP technique proposed in Section 6 can be incorporated with the solutions in Section 7 to efficiently monitor the global optimal locations in the event of updates.

At a high level, handling updates with FGP works as follows. We maintain a list of subgraphs obtained in Section 6 by FGP. Note that in Section 6, the FGP method may only compute the local optima for each edge in some of the subgraphs, which have larger upper bounds than others. We denote those subgraphs as $computed$ and $uncomputed$, those have not executed the local optima computation due to relatively smaller upper bounds. When there is an update, we compute the set of affected subgraphs and update their upper bounds. If an affected subgraph G_i is $computed$, we update the local optima for its edges by applying the algorithms in Section 7; otherwise, we keep it in the group of $uncomputed$. The update of upper bounds for all four types of update operation can be easily achieved according to Lemma 5. The next step is similar to the last step of FGP. Specifically, we examine the subgraphs in descending order: for $uncomputed$ subgraphs, we sort them by their upper bounds; for $computed$ subgraphs, we sort them by their local optima. For each examined G_i, if it is $uncomputed$, we apply the algorithm in Sections 4 and 5 to identify the local optima in G_i and move it to $computed$; if it is $computed$, we take its local optima. If the current global optima is larger than the local optima (upper bound for $uncomputed$ subgraphs) of the next subgraph, we terminate the search.

The benefit of incorporating FGP for updates is that we do not need to maintain the local optima for all edges; this saves both running time and memory consumption. However, a potential limitation with the above algorithm is that the number of $computed$ subgraphs will never decrease. This may lead to a situation where most subgraphs will become $computed$ after many update operations, which eliminates the benefit of FGP. To avoid this problem, we propose to control the number of $computed$ subgraphs since we need to maintain the local optima for all edges in $computed$ subgraphs. Specifically, once we obtain the global optima, we change those unexamined $computed$ subgraphs, whose upper bound is less than the global optima, to $uncomputed$.

9 Continuous Optimal Location Queries

Another important and interesting contribution made by our dynamic OLQ methods is the enabling of continuous optimal location queries. Note that in numerous applications in spatial databases, the continuous monitoring of query results is desirable, and sometimes a critical requirement. The reason is that spatial objects often are moving, and updates to their locations are fairly common in many applications. For example, extensive efforts have been devoted to this topic, such as continuous nearest neighbor queries [11, 19], continuous skyline computations [14, 18], and many others. To that end, we will show that our dynamic update techniques enable the efficient execution of continuous optimal location queries (continuous OLQ).

Consider the following application scenario. The city office wants to answer optimal location queries to different business users as where the optimal location is for them to build their new facility. Once an optimal location has been identified, a business user is like to go ahead and build a new facility at that location. Without our dynamic update methods, the city needs to answer the next optimal location query from another business user from scratch. However, with our dynamic monitoring techniques, the city office can now *continuously* answer a sequence of optimal location queries, amid a number of updates in between. Note that in this process, not only a new facility can be inserted, an existing facility may be removed. Furthermore, clients on the road network may also have changed their locations over a period of time. These challenges must be addressed by our dynamic monitoring techniques. We denote this problem as the *continuous optimal location queries*. Formally, a user supplies a *ordered sequence of operations*, where an operation could be an optimal location query (any one of three types OLQs that we have studied), or an updates on either facilities and/or clients. The goal is to be able to continuously produce answers to these optimal location queries in an incremental fashion. Between any two optimal location queries in the sequence, there could be a (large) number of update operations on both facilities and clients.

That said, our proposed dynamic methods can be easily adapted to answer continuous optimal location queries, where both clients and facilities may update their positions arbitrarily on the road network. We assume a client and/or a facility will issue an update with a new location (and an old location which is his/her current location on the network, if it is an existing client or facility), whenever a change has been made.

The first optimal location query from the sequence of operations can be answered using our static algorithms discussed in Sections 4, 5, and 6. Subsequently, we can continuously monitor the optimal locations for subsequent OLQs in the input sequence by treating any update operation as a deletion (if an old location exists) followed by an insertion (of either a client or a facility), which is done using the recomputation methods proposed in Sections 7 and 8.

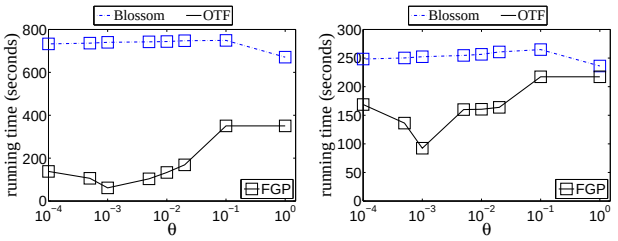
10 Experiments

This section experimentally evaluates the proposed solutions. For each type of OL queries, we examine two approaches for traversing the edges in E_c, (i) the *Basic* approach that computes the local optimal locations on every edge in E_c before returning the final results and (ii) the *Fine-Grained Partitioning (FGP)* approach. For each approach, we combine it with two different techniques for deriving attraction sets, i.e., *Blossom* and *OTF*. We implement our algorithms in C++, and perform all experiments on a Linux machine with an Intel Xeon 2GHz CPU and 4GB memory.

Our implementation uses the widely adopted road network representation proposed by Shekhar and Liu [25]. Besides the road network, in the dynamic scenario, we also maintain a list of local optimal locations and additional information for getting attraction sets. Specifically, for *Basic* approach, we maintain a list of all edges' local optimal locations; for *FGP* approach, we maintain a list of subgraphs, and each subgraph contains a list of local optimal locations for its edges. In addition, for *Blossom* method, we maintain the attraction sets for all vertices in G, while for *OTF* method, we maintain the nearest facility for each vertex in G as well as the distance between them.

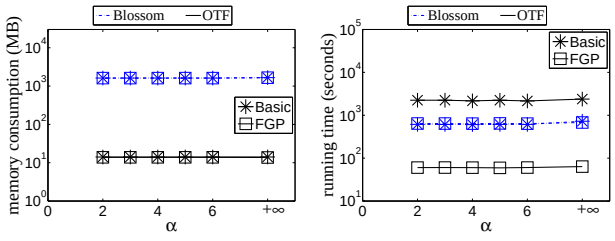
The running time reported in our OLQ computation experiments includes the cost of all algorithmic components of our framework, including the overhead for computing attractor distances using Erwig and Hagen's algorithm [7] (see Section 5). The running time for updates reports the cost after the OLQ computation.

Datasets. We use two real road network datasets, *SF* and *CA*, obtained from the *Digital Chart of the World Server*. In



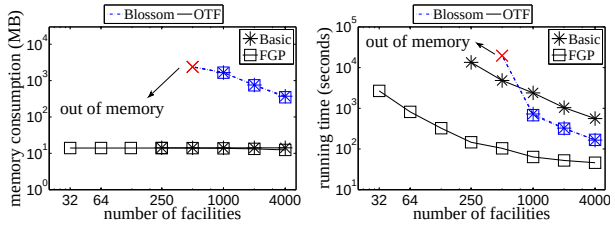
(a) *SF* (b) *CA*

Fig. 16 Running Time vs. θ (CLQ)



(a) Memory Consumption (b) Running Time

Fig. 17 Effect of α (CLQ on *SF*)



(a) Memory Consumption (b) Running Time

Fig. 18 Effect of $|F|$ (CLQ on *SF*)

particular, *SF* (*CA*) captures the road network in San Francisco (California), and contains 174,955 nodes and 223,000 edges (21,047 nodes and 21,692 edges). We obtain a large number of real building locations in San Francisco (California) from the *OpenStreetMap* project, and use random sample sets of those locations as facilities and clients on *SF* (*CA*). We synthesize the weight of each client by sampling from a Zipf distribution with a skewness parameter $\alpha > 1$.

We also generate a synthetic dataset(denoted as *UN*), which has *SF* as its underlying road network and facilities and clients in uniform distribution. For the facilities and clients which are not located on road network edges, we snapped them to the closest edge.

Default Settings. For the computation of all three types of OL queries, we vary five parameters in our experiments: (i) the number of facilities $|F|$, (ii) the number of clients $|C|$, (iii) the percentage $\tau = |E_c^\circ|/|E^\circ|$ of edges (in the given road network) where the new facility can be built, (iv) the input parameter θ of the *FGP* algorithm (see Figure 11), and (v) the skewness parameter α of the Zipf distribution from which we sample the weight of each client. For the updates, we vary two parameters: $|F|$ and $|C|$. For $AddC(c)$ and $DelC(c)$, we randomly generate 1000 updates, and for $AddF(f)$ and $DelF(f)$, we randomly generate 50 updates. Unless specified, we set $|F|$ =1000 and $|C|$ =300,000, so as to capture the likely scenario in practice where the number of clients is much larger than the number of facilities. We also set $\tau = 100\%$, in which case the OL queries are most computationally challenging, since we need to consider *every* point in the road network as a candidate location. The default value of θ is set to 100% (1‰) when *Blossom* (*OTF*) is used to derive attraction sets, as discussed in Section 6.2. Finally, we set $\alpha = +\infty$ by default, in which case all clients have a weight 1.

10.1 The OLQ Computation in the Static Case

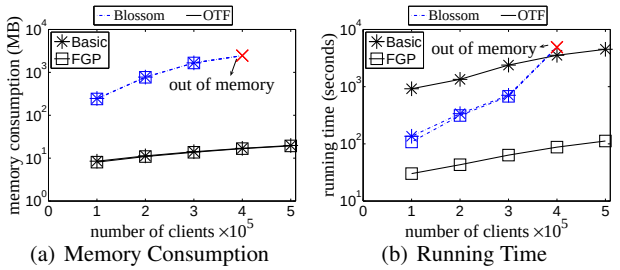
We first investigate the efficiency and scalability of various methods for different OLQ queries in the static case.

Effect of θ. Our first sets of experiments focus on competitive location queries (CLQ). Figure 16 shows the effect of θ on the running time of our solutions that incorporate *FGP*. Observe that, when *Blossom* is adopted to compute attraction sets, our solution is most efficient at $\theta = 1$. This is consistent with the analysis in Section 6.2 that $\theta = 1$ (i) leads to the tightest benefit upper-bounds, and thus, (ii) facilitates early termination of edge traversal. In contrast, when *OTF* is adopted, $\theta = 1$ results in inferior computational efficiency. This is because, when *OTF* is employed, a larger θ leads to a higher cost for deriving benefit upper-bounds, which offsets the efficiency gain obtained from the tighter upper-bounds. On the other hand, $\theta = 1‰$ strikes a good balance between the overhead of upper-bound computation and the tightness of the upper-bounds, which justifies our choice of default values for θ. Note that another possible way of identifying a good θ value is to run tests on a (small) random sample of the data set on the given network.

The *OTF*-based solution outperforms the *Blossom*-based approach in both cases. The reason is that, regardless of the value of θ, the *Blossom*-based approach requires computing the attraction sets of all vertices. In contrast, the *OTF*-based solution only needs to derive the attraction sets of the vertices in each *subgraph* it visits. Since the *OTF*-based solution visits only subgraphs whose benefit upper-bounds are large, it computes a much smaller number of attraction sets than the *Blossom*-based approach does, and hence, it achieves superior efficiency.

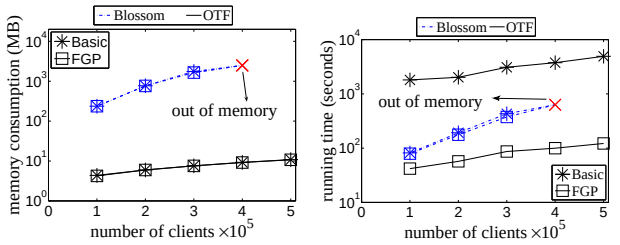
For brevity, in the following we focus on the larger *SF* dataset. The results on *CA* are qualitatively similar.

Effect of α. Figure 17 shows the effect of α on the performance of our solutions, varying α from 2 to $+\infty$. Evidently, the memory consumption and running time of our solutions are insensitive to the clients' weight distributions. The reason is that, for *Basic* approach, the change of α does not change the complexity of our solution. And for *FGP* ap-



(a) Memory Consumption (b) Running Time

Fig. 19 Effect of $|C|$ (CLQ on *SF*)



(a) Memory Consumption (b) Running Time

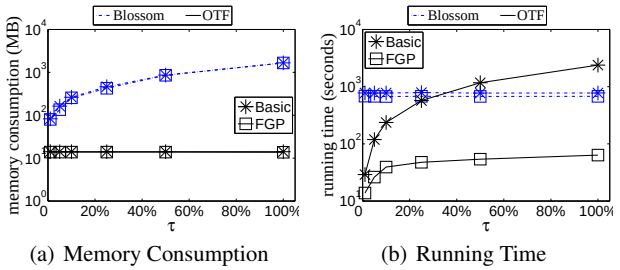
Fig. 20 Effect of $|C|$ (CLQ on *UN*)

proach, the distribution of weights is independent with the clients' locations, so the distribution of benefit upper bounds of subgraphs will not be affected by α, which indicates that the performance of *FGP* approach is also insensitive to the weight distribution.

Effect of $|F|$. Next, we vary the number of facilities ($|F|$) from 32 to 4000. Figures 18(a) illustrates the memory consumption of our solutions on *SF*. The methods based on *Blossom* incur significant space overheads, and run out of memory when $|F|$ <1000. This is due to the $O(n^2)$ space complexity of *Blossom*. In contrast, the memory consumption of *OTF*-based methods are always below 20MB, since *OTF* incurs only $O(n)$ space overhead.

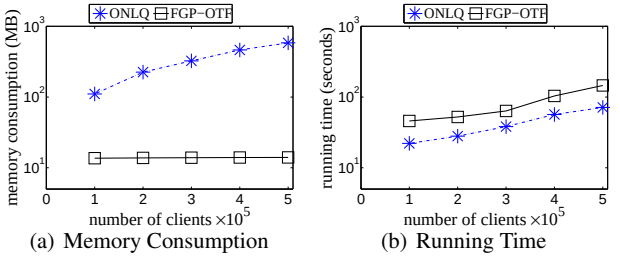
Figure 18(b) shows the running time of each of our solutions as a function of $|F|$. The methods that incorporate *FGP* outperform the approaches with *Basic* in all cases, since *FGP* provides much more effective means to avoid visiting the edges that do not contain optimal locations. In addition, when *FGP* is adopted, the *OTF*-based approach is superior to the *Blossom*-based approach, as is consistent with the results in Figure 16. Finally, the running time of all solutions decreases with the increase of $|F|$. The reason is that, when $|F|$ is large, each client c tends to have a smaller attractor distance $a(c)$. This reduces the number of road network vertices that are within $a(c)$ distance to c, and hence, c should appear in a smaller number of attraction sets. Therefore, the attract set of each vertex in the road network would become smaller, in which case that *Blossom* and *OTF* can be executed in shorter time. Consequently, the overall running time of our solutions is reduced.

Effect of $|C|$. The next set of experiments investigate the



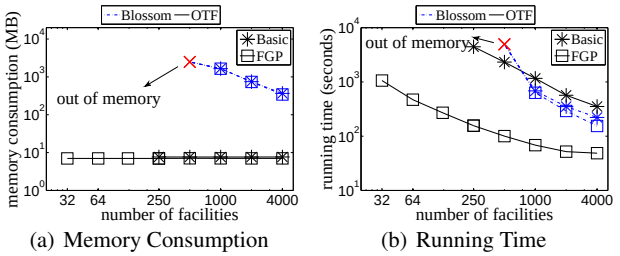
(a) Memory Consumption (b) Running Time

Fig. 21 Effect of τ (CLQ on *SF*)



(a) Memory Consumption (b) Running Time

Fig. 22 Comparison with ONLQ in [10] (on *SF*)

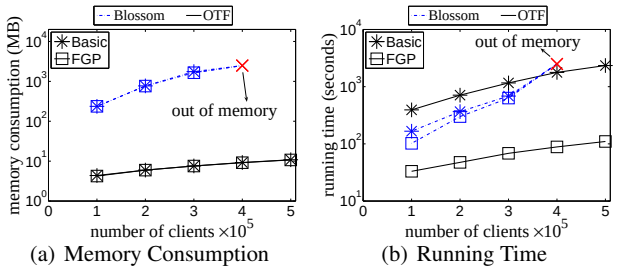


(a) Memory Consumption (b) Running Time

Fig. 23 Effect of $|F|$ (MinSumLQ on *SF*)

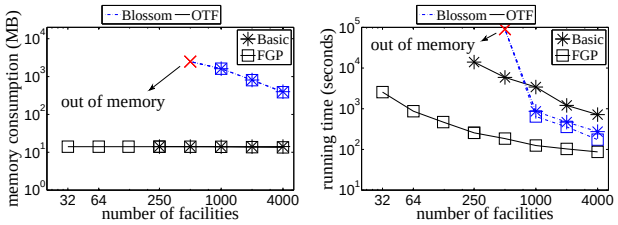
scalability of our solutions by varying the number of clients, $|C|$, from 100,000 to 500,000. Figures 19(a) (19(b)) shows the space consumption (running time) of our solutions as functions of $|C|$. As in the previous experiments, *Blossom*-based approaches consume enormous amounts of memory, while the method that incorporates both *FGP* and *OTF* consistently outperform all the other methods in terms of both space and time. The running time of all solutions increases with $|C|$, because a larger $|C|$ leads to (i) more edges in the transformed network G and (ii) more clients in each attraction set, both of which complicate the computation of optimal locations. We also conduct the same experiment on *UN* dataset. Figure 20 shows the result. As we can see, the data distribution does not affect the performance of the proposed methods. Especially, with a uniform spatial distribution of facilities and clients, our FGP approach still has the best performance. That is because, the sizes of subgraphs generated by FGP approach are different, so the upper bounds of different subgraphs can be various, which indicates that the FGP approach is still effective when the spatial distribution of facilities and clients is uniform.

Effect of τ. Figure 21 illustrates the memory consumption and running time of our solutions when τ changes from 1% to 100%. *Blossom*-based approaches require less memory



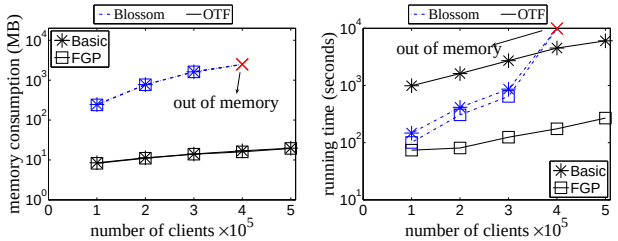
(a) Memory Consumption (b) Running Time

Fig. 24 Effect of $|C|$ (MinSumLQ on *SF*)



(a) Memory Consumption (b) Running Time

Fig. 25 Effect of $|F|$ (MinMaxLQ on *SF*)

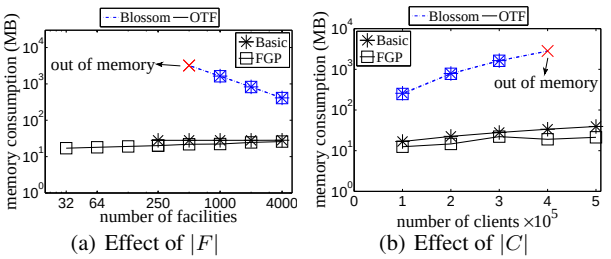


(a) Memory Consumption (b) Running Time

Fig. 26 Effect of $|C|$ (MinMaxLQ on *SF*)

when τ decreases, since (i) a smaller τ leads to fewer edges in E_c and (ii) *Blossom* stores only the attraction sets of the endpoints of the edges in E_c. In contrast, the space overheads of *OTF*-based methods do not change with τ, since they always compute attraction sets on the fly, regardless of the value of τ.

On the other hand, the running time of *Blossom*-based approaches is not affected by τ. This is because, *Blossom* computes attraction sets, by invoking Dijkstra's algorithm for each client $c \in C$, and putting c in the attraction set of every vertex $v \in V$ such that $d(c, v) \leq a(c)$. Observe that, even if we require only a single attraction set of a vertex $v_0 \in V$, *Blossom* still needs to execute Dijkstra's algorithm once for each client c; otherwise, it is impossible to decide whether $d(c, v_0) \leq a(c)$ holds or not. As a consequence, the efficiency of *Blossom*-based approaches does not improve with the decrease of τ. In contrast, *OTF*-based solutions incur much less computation overhead when τ is reduced, since they compute attraction sets by invoking *OTF* only on the vertices of the edges in E_c. The decrease in τ renders $|E_c|$ smaller, in which case the *OTF*-based solutions require fewer executions of the *OTF* algorithm, and hence, their running time decreases.



(a) Effect of $|F|$ (b) Effect of $|C|$

Fig. 27 Memory Consumption of $AddC(c)$ (CLQ on *SF*)

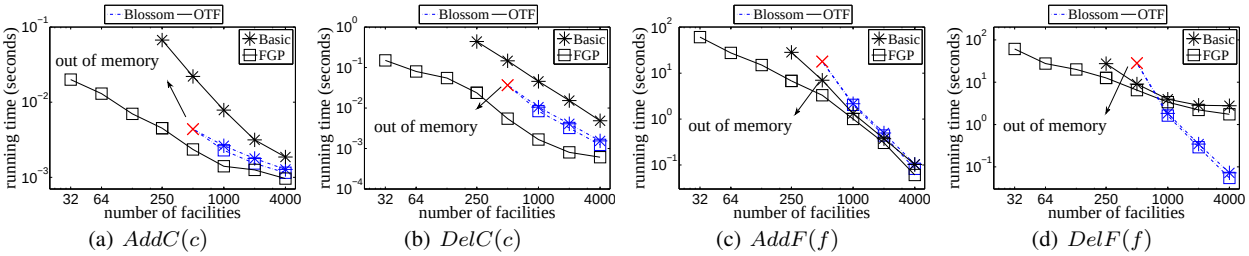
Comparison with method in [10]. Figure 22 compares the memory consumption and running time of our solution with the method proposed in [10], which is denoted as ONLQ for convenience. In order to compare the performance fairly, we re-implemented ONLQ using C++, and perform all experiments on a Linux machine with the previously mentioned configurations. Our (the OTF with FGP) method has a less memory consumption but longer running time compared to the ONLQ method when we vary $|C|$. This observation can also be found when we vary $|F|$.

MinSum and MinMax Location Queries. The rest of our experiments evaluate the performance of our solutions for MinSum location queries (MinSumLQ) and MinMax location queries (MinMaxLQ). In general, the experimental results are mostly similar to those for competitive location queries. This is not surprising, because our solutions for the three types of queries follow the same framework, and adopt the same algorithmic components (e.g., *Blossom* and *OTF*).

Figures 23 and 24 show the effects of $|F|$ and $|C|$ on the performance of our solutions for MinSum location queries. Again, the method that combines *FGP* and *OTP* achieves the best space and time efficiency in all cases. In addition, *Blossom*-based approaches entails excessively high memory consumption, especially when $|F|$ <1000 or $|C|$ >300,000.

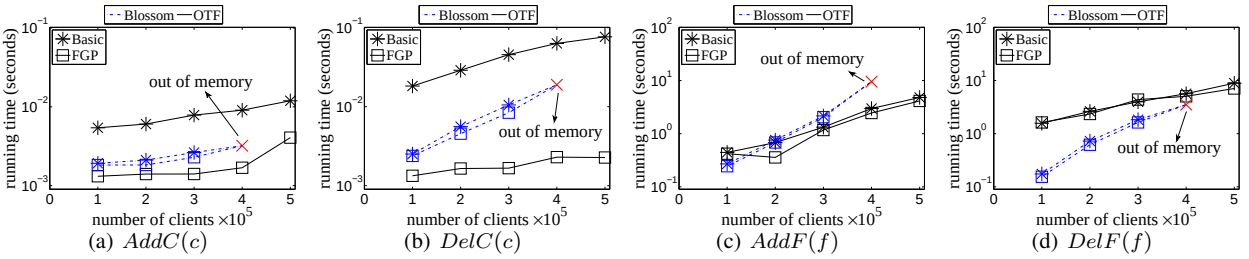
Figures 25 and 26 plot the memory consumption and running time of our solutions for MinMax location queries. The relative performance of each method remains the same as in Figures 23 and 24. Interestingly, each method incurs a higher computation time for MinMax location queries than for the other two types of OL queries. This is caused by the fact that, our solutions identify local MinMax locations on any given edge e, by computing the upper envelope of a set of WAD functions (see Section 4.3). This procedure is more costly than computing the local competitive (MinSum) locations on e.

Summary. Our results show that the solutions incorporating *FGP* and *OTF* consistently achieve the best performance for all three types of OL queries, in term of both space and time. In particular, they require less than 20MB memory and 200 seconds to answer OL queries in a road network with 174,955 nodes, 223,000 edges, up to 500,000 clients, and down to 32 facilities (recall that the performance of the so-



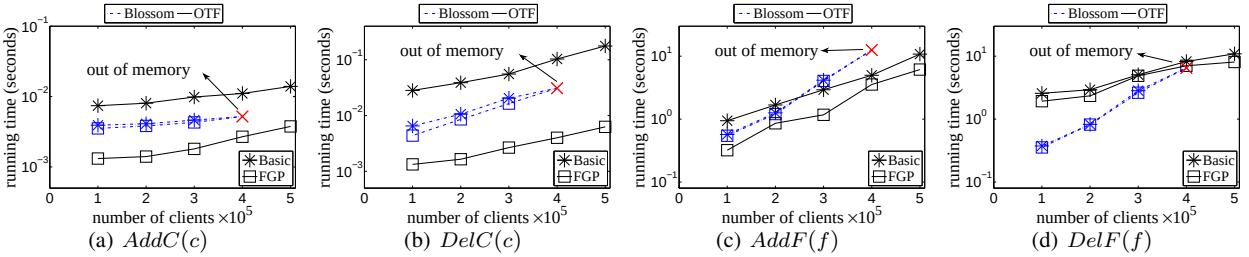
(a) $AddC(c)$ (b) $DelC(c)$ (c) $AddF(f)$ (d) $DelF(f)$

Fig. 28 Update running time: effect of $|F|$ (CLQ on *SF*)



(a) $AddC(c)$ (b) $DelC(c)$ (c) $AddF(f)$ (d) $DelF(f)$

Fig. 29 Update running time: effect of $|C|$ (CLQ on *SF*)



(a) $AddC(c)$ (b) $DelC(c)$ (c) $AddF(f)$ (d) $DelF(f)$

Fig. 30 Update running time: effect of $|C|$ (CLQ on *UN*)

lutions improve with the number of facilities). We also recommend that setting $\theta = 1‰$ for these solutions, so as to optimize the overall running time.

10.2 Incremental OLQ Queries Under Updates

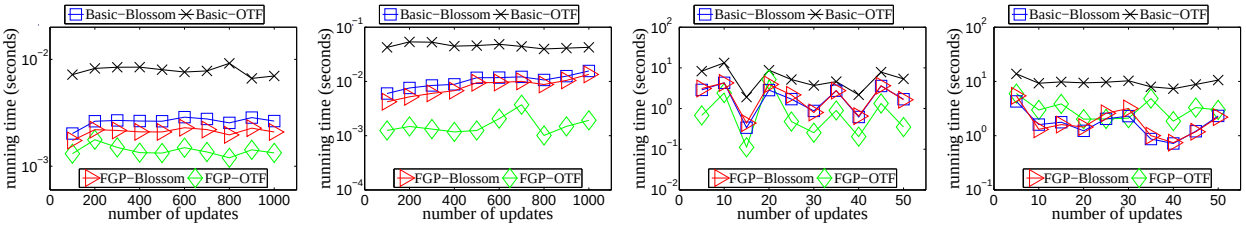
We next examine the impacts of updates and the efficiency of our incremental methods for answering OLQ queries in an event of update. As shown in default settings, in each experiment, we continuously add or delete 1000 clients or 50 facilities, and we show the average running time unless otherwise specified.

Effect of $|F|$. The first set of experiments focuses on competitive location queries (CLQ). We vary the number of facilities ($|F|$) from 32 to 4000. Figure 27(a) shows the memory consumption of $AddC(c)$ as a function of $|F|$. Clearly, *Blossom*-based methods consume enormous amounts of memory due to the $O(n^2)$ space overhead for maintaining the attraction sets of all vertices. For *OTF*-based methods, the memory consumption is slightly larger than that of the OLQ computation in the static case due to the maintaining of local optimal locations, but still very small. The memory consumption in other three operations have similar trends, so we omit them for brevity.

Figure 28 shows that the running time of all solutions decreases with the increase of $|F|$. The reason is that, when $|F|$ is larger, a client will have a smaller attractor distance and a facility will attract a smaller set of clients. This reduces the effect range of a client and the computation cost when the optimal locations on an edge need to be recomputed. Furthermore, the running time of an update is much (at least 2 to 3 orders of magnitude) less than that of the OLQ computation from Section 10.1. That is because each update only takes effect on a part of the road network in all of the proposed incremental update algorithms.

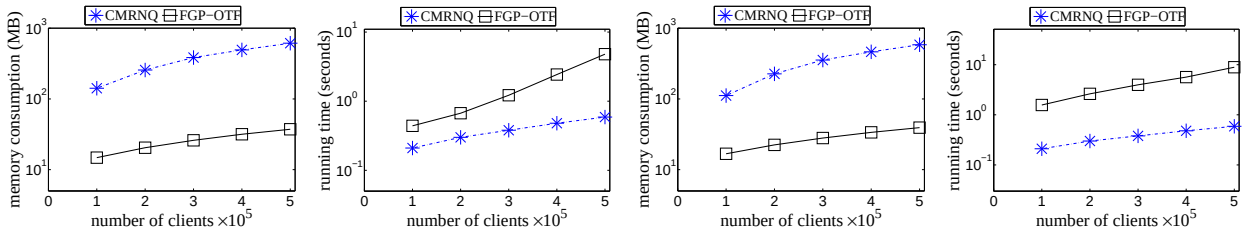
In particular, Figures 28(a) and 28(b) show the running time of the $AddC(c)$ and $DelC(c)$ operations. Clearly, the *FGP* approaches always outperform the *Basic* approaches, since with *FGP* we do not need to maintain local optimal locations for all edges. Moreover, the method that does incorporate *FGP* and *OTF* outperforms the other three approaches.

Figures 28(c) and 28(d) show the running time for the $AddF(f)$ and $DelF(f)$ operations. In this case, the *FGP* approaches are quite similar to the *Basic* approaches. That is because adding or deleting a facility will cause the changes



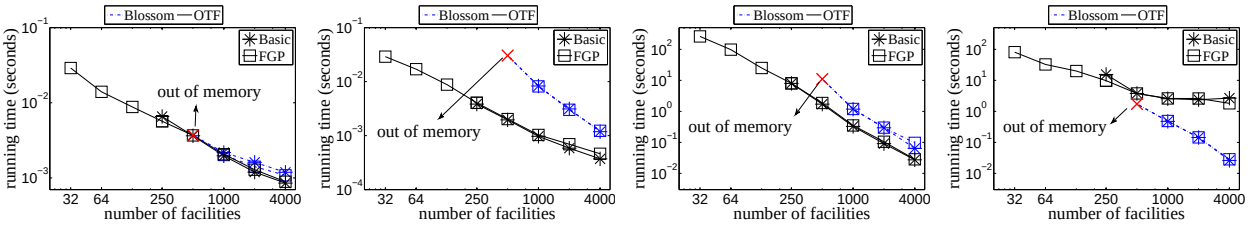
(a) $AddC(c)$ (b) $DelC(c)$ (c) $AddF(f)$ (d) $DelF(f)$

Fig. 31 Update running time: growth of updates (CLQ on *SF*)



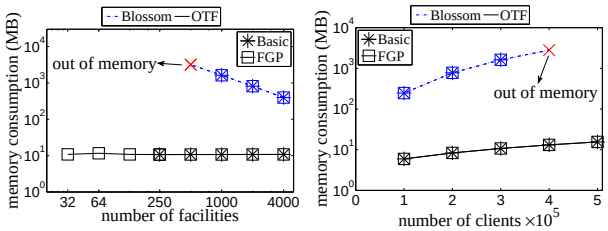
(a) AddF(f) (b) AddF(f) (c) $DelF(f)$ (d) $DelF(f)$

Fig. 32 Comparison with CMRNQ in [11] (on *SF*)



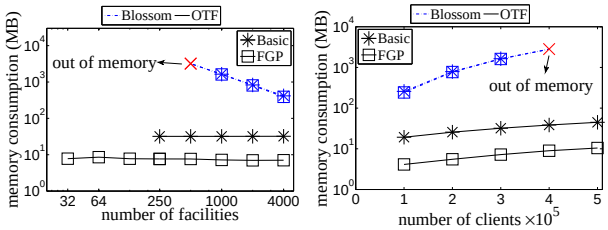
(a) $AddC(c)$ (b) $DelC(c)$ (c) $AddF(f)$ (d) $DelF(f)$

Fig. 35 Update running time: effect of $|F|$ (MinSumLQ on *SF*)



(a) Effect of $|F|$ (b) Effect of $|C|$

Fig. 33 Memory Consumption of $AddC(c)$ (MinSumLQ on *SF*)



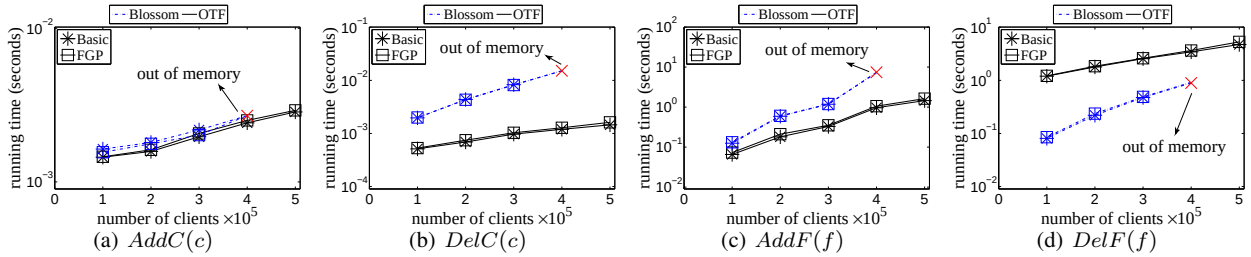
(a) Effect of $|F|$ (b) Effect of $|C|$

Fig. 34 Memory Consumption of $AddC(c)$ (MinMaxLQ on *SF*)

of attractor distances for many clients, which may lead to the change of the upper bounds for many subgraphs. As a result, the amount of edges that need to be maintained will be large and there is a relatively high chance that at least one previously *uncomputed* subgraph will be fully computed during the update operation, which may cause a high computation overhead. The *OTF*-based methods slightly outperform the *Blossom*-based methods in $AddF(f)$, but are much slower than *Blossom*-based methods in $DelF(f)$. That is because, in *OTF*-based methods, we need to maintain the distance from each vertex to its nearest facility (see Section 5) and when deleting a facility f, we must find the new nearest facility for each of the vertices that previously take f as its nearest facility, which takes $O(n \log n)$ time and is a significant computation overhead for updates.

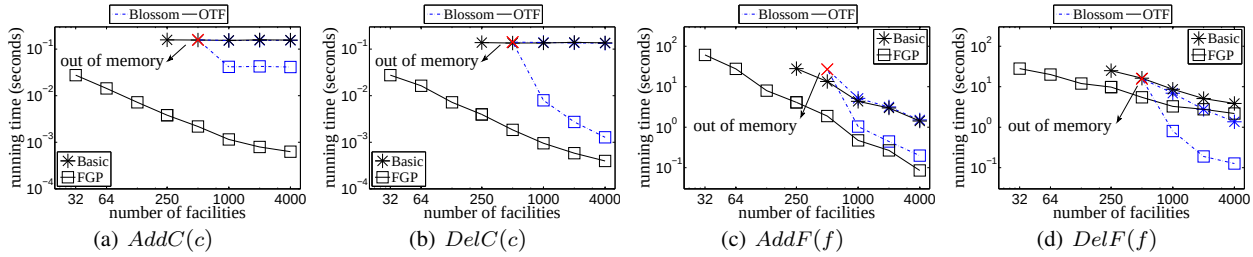
Effect of $|C|$. Next, we vary the number of clients ($|C|$) from 100,000 to 500,000. Figure 27(b) shows the memory consumption of $AddC(c)$ as a function of $|C|$. Same as the discussion for the effect of $|F|$, the memory consumption is also similar to that in the static OLQ computation. Similar trends were also observed for the memory consumption of other three types of updates, which are omitted for brevity.

Figure 29 shows the running time as a function of $|C|$. The running time of all solutions increases as $|C|$ goes up. That is because, when $|C|$ is large, $|E|$ is also large and there are more clients in each attraction set, which will increase the computation complexity. Nevertheless, all incremental



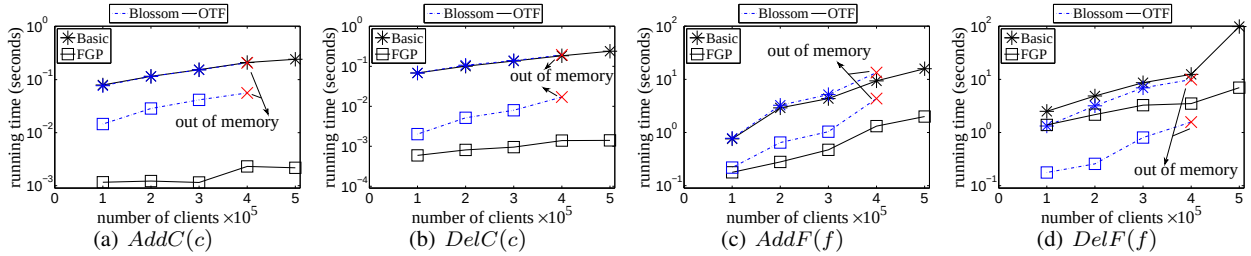
(a) $AddC(c)$ (b) $DelC(c)$ (c) $AddF(f)$ (d) $DelF(f)$

Fig. 36 Update running time: effect of $|C|$ (MinSumLQ on *SF*)



(a) $AddC(c)$ (b) $DelC(c)$ (c) $AddF(f)$ (d) $DelF(f)$

Fig. 37 Update running time: effect of $|F|$ (MinMaxLQ on *SF*)



(a) $AddC(c)$ (b) $DelC(c)$ (c) $AddF(f)$ (d) $DelF(f)$

Fig. 38 Update running time: effect of $|C|$ (MinMaxLQ on *SF*)

methods under different update operations are much more efficient when compared to computing OLQs from scratch using the static OLQ methods from Section 10.1.

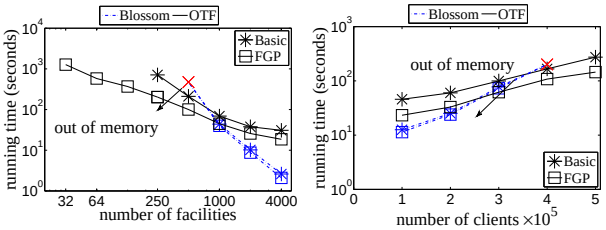
Figure 30 shows the effect of $|C|$ on *UN* dataset. As we can see, the trends are similar to those in Figure 29. These figures shows that the proposed methods are insensitive to the distribution of facilities and clients. The same conclusion is observed for other variants of OLQ.

Effect of growth of updates. Next we investigate the stability of our solution when updates grows over time. Figure 31 shows the running time of each update operation in a series of continuous updates (1000 clients updates and 50 facilities updates). In Figures 31(a) and 31(b), we show the average running time of every continuous 100 clients updates. In Figures 31(c) and 31(d), we show the average running time of every continuous 5 facilities updates. It can be seen that the running time for all four update operations is stable with the growth of number of updates. For basic approach, it is because we maintain local optimal locations for all edges and the time of updates will not affect anything in the framework. For FGP approach, since we develop a method to avoid the number of subgraphs that need to be maintained to become larger and larger with more updates, as discussed in the last paragraph in Section 8, the updating time is still stable.

Comparison with method in [11]. Figure 32 compares the memory consumption and running time of the OTF-FGP method with the method proposed in [11], which is denoted as CMRNQ for convenience. The OTF-FGP method has a less memory consumption but longer running time compared to the CMRNQ method when we vary $|C|$. We omit the figures for varying $|F|$ as they have the similar trends.

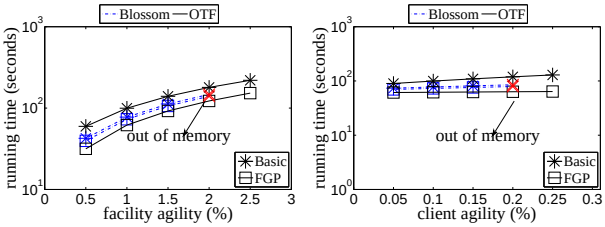
MinSum and MinMax Location Queries. The rest of our experiments evaluates the performance of proposed incremental solutions on updates for MinSumLQ and MinMaxLQ queries. These results are shown in Figures 33- 38, and they generally report similar trends as that in the CLQ problem discussed above.

In particular, Figures 33 and 34 show that our best incremental update methods (i.e., the ones that incorporate both FGP and OTF) for MinSumLQ and MinMaxLQ enjoy small memory footprints for the $AddC(c)$ operation. The memory consumption of these methods for other three types of update operations is similar, and were omitted for brevity.



(a) Effect of $|F|$ (b) Effect of $|C|$

Fig. 39 COLQ Running time: effect of $|F|$ and $|C|$ (CLQ on *SF*)



(a) Effect of f_F (b) Effect of f_C

Fig. 40 COLQ Running time: effect of agility (CLQ on *SF*)

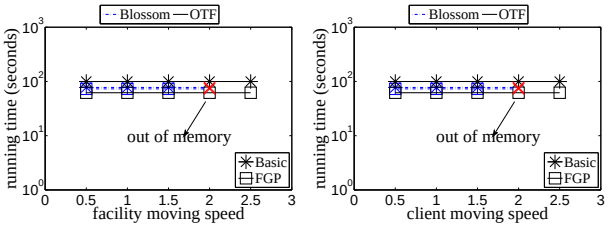
Figures 35 and 36 investigate the effect of $|F|$ and $|C|$ on the running time under different types of update operations for the MinSumLQ queries, respectively. In general, larger $|F|$ leads to smaller update cost and larger $|C|$ on the other hand introduces higher update cost, for similar reasons as we have explored for the CLQ queries. Nevertheless, compared to the running time of the corresponding static OLQ methods from Sections 10.1, our incremental update methods are much more efficient (faster by at least 3-4 orders of magnitude).

Similar experiments were carried out in Figures 37 and 38, to investigate the effect of $|F|$ and $|C|$ on the running time under different types of update operations for the MinSumLQ queries, respectively. And not surprisingly, we have observed similar trends, where the incremental update methods' running time are at least 3-4 orders of magnitude smaller compared to that in the naive approach (if we were to run the static OLQ computations from scratch after every update using the static methods from Section 10.1).

Summary. Our results show that the incremental update methods are highly effective, efficient, and scalable. In particular, solutions incorporating both *FGP* and *OTF* achieve the best performance for all three types of OL queries on all update operations. Last but not least, our framework for handling updates can incrementally answer and monitor OL queries in a much faster fashion than the baseline approach of recomputing the OLs from scratch after each update using the static methods from Section 10.1.

10.3 Continuous Optimal Location Queries

In this section, we investigate the performance of adapting our update algorithms to answer the continuous optimal location queries (COLQ). We set up the sequence of operations in a continuous optimal location query as follows.



(a) Effect of d_F (b) Effect of d_C

Fig. 41 COLQ Running time: effect of moving distance (CLQ on *SF*)

In-between any two optimal location queries, a percentage f_F (f_C) of the facilities (clients) will change their locations, where f_F (f_C) is called the facility (client) agility. For any facility or client that is selected to issue an update, its new location is given as follows. This facility or client will perform a random walk on the road network with a fixed distance d_F (d_C) to determine its new location.

That said, here we only showed the results of the CLQ variant (competitive location query). The results for the other two types of optimal location queries, MinSumLQ and MinMaxLQ, have exhibited similar trends. Hence, we omitted them for brevity.

In the following experiments, the default values of $|F|$ and $|C|$ is the same as that in the previous experiments, i.e. $1,000$ and $300,000$ respectively. The f_F (f_C) is set to 1% (0.1%) by default, , which means that 10 facilities and 300 clients will issue updates between two OLQs. The default value of d_F (d_C) is set to the average edge length. We measured the average running time of answering every OLQ query from a continuous OLQ.

Figure 39(a) examines the effect of the $|F|$. With the increasing number of facilities, the running time for the COLQ has decreased. When $|F| = 4,000$, the COLQ can be answered in several seconds using the blossom based methods under the default setup. In this case, more facilities imply that the attraction distance of each client will be reduced, leading to faster dynamic update time. On the other hand, the running time has increased with more clients, as shown in Figure 39(b). Note that, the update time for each client is almost constant. The reason for the increase of query time in Figure 39(b) is because that we set the client agility to a percentage of $|C|$, i.e. there are more clients moving when $|C|$ becomes larger.

Figure 40(a) shows that the running time of COLQ increases linearly to the facility agility. But the running time only increases slightly when we increase the client agility as shown in Figure 40(b). This is because that the running time of the facility update operations is the dominating factor, although there are more client updates in each experiment.

Finally, we vary the moving distance of the facilities and clients. As we can see in Figure 41, the running time is not

affected by either d_F or d_C. This is because that our method answers COLQ for each facility (client) by a deletion and an insertion, which are insensitive to the moving distance of the facility (client).

11 Conclusion

This work presents a comprehensive study on optimal location queries in large, disk-resident road network databases, closing the gap between previous studies and practical applications in road networks. Our study covers three important types of optimal location queries, and introduces a unified framework that addresses all three query types efficiently. We have also extended our framework to handle updates efficiently in an incremental fashion. Extensive experiments on real datasets demonstrate the scalability of our solution in terms of running time and space consumption. An interesting direction for future work includes the optimal location queries for moving objects in road networks.

Acknowledgment

Bin Yao was supported by the NSFC (Grant No. 61202025), the 863 Program of China (Grant No. 2011AA01A202), the Program for Changjiang Scholars and Innovative Research Team in University of China (IRT1158, PCSIRT), Shanghai Excellent Academic Leaders Plan (No. 11XD1402900), Microsoft Research Asia (the Urban Informatics Research Grant), and the STCSM (Grant No. 12ZR1414900). Xiaokui Xiao was supported by the Nanyang Technological University under SUG Grant M58020016, and by Microsoft Research Asia under an Urban Informatics Research Grant. Feifei Li was supported in part by NSF Grant IIS-1251019.

References

1. Berg, M., Cheong, O., van Kreveld, M., Overmars, M.: Computational Geometry: Algorithms and Applications, 3rd edn. Springer-Verlag (2008)
2. Cabello, S., Díaz-Báñez, J.M., Langerman, S., Seara, C., Ventura, I.: Reverse facility location problems. In: Proc. of the Canadian Conference on Computational Geometry (CCCG), pp. 68–71 (2005)
3. Chen, Z., Shen, H.T., Zhou, X., Yu, J.X.: Monitoring path nearest neighbor in road networks. In: Proc. of ACM Management of Data (SIGMOD), pp. 591–602 (2009)
4. Deng, K., Zhou, X., Shen, H.T., Sadiq, S., Li, X.: Instance optimal query processing in spatial networks **18**(3), 675–693 (2009)
5. Dijkstra, E.W.: A note on two problems in connexion with graphs. Numerische Mathematik **1**, 269–271 (1959)
6. Du, Y., Zhang, D., Xia, T.: The optimal-location query. In: Proc. of Symposium on Advances in Spatial and Temporal Databases (SSTD), pp. 163–180 (2005)
7. Erwig, M., Hagen, F.: The graph voronoi diagram with applications. Networks **36**, 156–163 (2000)
8. Farahani, R.Z., Hekmatfar, M.: Facility Location: Concepts, Models, Algorithms and Case Studies, 1st edn. Physica-Verlag HD (2009)
9. Fotakis, D.: Incremental algorithms for facility location and k-median. Theor. Comput. Sci. **361**(2-3), 275–313 (2006)
10. Ghaemi, P., Shahabi, K., Wilson, J., Banaei-Kashani, F.: Optimal network location queries. In: Proc. of ACM Symposium on Advances in Geographic Information Systems (GIS) (2010)
11. Ghaemi, P., Shahabi, K., Wilson, J., Banaei-Kashani, F.: Continuous maximal reverse nearest query on spatial networks. In: Proc. of ACM Symposium on Advances in Geographic Information Systems (GIS) (2012)
12. Ghaemi, P., Shahabi, K., Wilson, J., Banaei-Kashani, F.: A comparative study of two approaches for supporting optimal network location queries. GeoInformatica **17**(2) (2013)
13. Hershberger, J.: Finding the upper envelope of n line segments in $o(n \log n)$ time. Inf. Process. Lett. **33**(4), 169–174 (1989)
14. Huang, Z., Lu, H., Ooi, B.C., Tung, A.K.H.: Continuous skyline queries for moving objects. IEEE Trans. Knowl. Data Eng. **18**(12), 1645–1658 (2006)
15. Jensen, C.S., Kolářvr, J., Pedersen, T.B., Timko, I.: Nearest neighbor queries in road networks. In: Proc. of ACM Symposium on Advances in Geographic Information Systems (GIS), pp. 1–8 (2003)
16. Kolahdouzan, M.R., Shahabi, C.: Voronoi-based k-nearest neighbor search for spatial network databases. In: Proc. of Very Large Data Bases (VLDB), pp. 840–851 (2004)
17. Meyerson, A.: Online facility location. In: Symposium on Foundations of Computer Science (FOCS), pp. 426–431 (2001)
18. Morse, M.D., Patel, J.M., Grosky, W.I.: Efficient continuous skyline computation. In: ICDE, p. 108 (2006)
19. Mouratidis, K., Yiu, M.L., Papadias, D., Mamoulis, N.: Continuous nearest neighbor monitoring in road networks. In: Proc. of Very Large Data Bases (VLDB), pp. 43–54 (2006)
20. Nickel, S., Puerto, J.: Location Theory: A Unified Approach, 1st edn. Springer (2005)
21. Papadias, D., Zhang, J., Mamoulis, N., Tao, Y.: Query processing in spatial network databases. In: Proc. of Very Large Data Bases (VLDB), pp. 802–813 (2003)
22. Samet, H., Sankaranarayanan, J., Alborzi, H.: Scalable network distance browsing in spatial databases. In: Proc. of ACM Management of Data (SIGMOD), pp. 43–54 (2008)
23. Sankaranarayanan, J., Samet, H.: Distance oracles for spatial networks. In: Proc. of International Conference on Data Engineering (ICDE), pp. 652–663 (2009)
24. Sankaranarayanan, J., Samet, H., Alborzi, H.: Path oracles for spatial networks. PVLDB **2**(1), 1210–1221 (2009)
25. Shekhar, S., Liu, D.R.: CCAM: A connectivity-clustered access method for networks and network computations. IEEE Transactions on Knowledge and Data Engineering (TKDE) **9**(1), 102–119 (1997)
26. Wong, R.C.W., Özsu, T., Yu, P.S., Fu, A.W.C., Liu, L.: Efficient method for maximizing bichromatic reverse nearest neighbor. Proc. of the VLDB Endowment (PVLDB) **2**(1), 1126–1137 (2009)
27. Xiao, X., Yao, B., Li, F.: Optimal location queries in road network databases. In: ICDE, pp. 804–815 (2011)
28. Yiu, M.L., Mamoulis, N., Papadias, D.: Aggregate nearest neighbor queries in road networks. IEEE Transactions on Knowledge and Data Engineering (TKDE) **17**(6), 820–833 (2005)
29. Zhang, D., Du, Y., Xia, T., Tao, Y.: Progressive computation of the min-dist optimal-location query. In: Proc. of Very Large Data Bases (VLDB), pp. 643–654 (2006)