Symbols

Our favorite **list-of-sym** program:

```
; eat-apples : list-of-sym -> list-of-sym
(define (eat-apples l)
  (cond
   [(empty? l) empty]
   [(cons? l)
    (local [(define ate-rest (eat-apples (rest l)))]
      (cond
        [(symbol=? (first l) 'apple) ate-rest]
        [else (cons (first l) ate-rest)]))]))
```

- How about **eat-bananas**?
- How about **eat-non-apples**?

Symbols

Our favorite **`list-of-sym`** program:

```
; eat-apples : list-of-sym -> list-of-sym
(define (eat-apples l)
  (cond
   [(empty? l) empty]
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    (local [(define ate-rest (eat-apples (rest l)))]
      (cond
        [(symbol=? (first l) 'apple) ate-rest]
        [else (cons (first l) ate-rest)]))]))
```

- How about **`eat-bananas`**?
- How about **`eat-non-apples`**?

We know where this leads...

Filtering Symbols

```
; filter-syms : (sym -> bool) list-of-sym
; -> list-of-sym
(define (filter-syms PRED l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r
                (filter-syms PRED (rest l)))]
       (cond
         [(PRED (first l))
          (cons (first l) r)]
         [else r])))]))
```

Filtering Symbols

```
; filter-syms : (sym -> bool) list-of-sym
; -> list-of-sym
(define (filter-syms PRED l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r
               (filter-syms PRED (rest l)))]
       (cond
         [(PRED (first l))
          (cons (first l) r)]
         [else r])))]))
```

This looks *really* familiar

Last Time: Filtering Numbers

```
; filter-nums : (num -> bool) list-of-num
; -> list-of-num
(define (filter-nums PRED l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r
                (filter-nums PRED (rest l)))]
       (cond
         [(PRED (first l))
          (cons (first l) r)]
         [else r]))]))
```

Last Time: Filtering Numbers

```
; filter-nums : (num -> bool) list-of-num
; -> list-of-num
(define (filter-nums PRED l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r
               (filter-nums PRED (rest l)))]
       (cond
         [(PRED (first l))
          (cons (first l) r)]
         [else r])))]))
```

How do we avoid cut and paste?

Filtering Lists

We know this function will work for both number and symbol lists:

```
; filter : ...
(define (filter PRED l)
  (cond
    [(empty? l) empty]
    [(cons? l)
     (local [(define r
               (filter PRED (rest l)))]
       (cond
         [(PRED (first l))
          (cons (first l) r)]
         [else r])))]))
```

But what is its contract?

The Contract of Filter

How about this?

```
(num-OR-sym -> bool) list-of-num-OR-list-of-sym
-> list-of-num-OR-list-of-sym
```

```
; A num-OR-sym is either
;   - num
;   - sym

; A list-of-num-OR-list-of-sym is either
;   - list-of-num
;   - list-of-sym
```

The Contract of Filter

How about this?

```
(num-OR-sym -> bool) list-of-num-OR-list-of-sym
-> list-of-num-OR-list-of-sym
```

This contract is too weak to define **eat-apples**

```
; eat-apples : list-of-sym -> list-of-sym
(define (eat-apples l)
  (filter not-apple? l))

; not-apple? : sym -> bool
(define (not-apple? s)
  (not (symbol=? s 'apple)))
```

eat-apples must return a **list-of-sym**, but by its contract, **filter** might return a **list-of-num**

The Contract of Filter

How about this?

```
(num-OR-sym -> bool) list-of-num-OR-list-of-sym
-> list-of-num-OR-list-of-sym
```

This contract is too weak to define **`eat-apples`**

```
; eat-apples : list-of-sym -> list-of-sym
(define (eat-apples l)
  (filter not-apple? l))

; not-apple? : sym -> bool
(define (not-apple? s)
  (not (symbol=? s 'apple)))
```

`not-apple?` only works on symbols, but by its contract **`filter`** might give it a **`num`**

The Contract of Filter

The reason **`filter`** works is that if we give it a **`list-of-sym`**, then it returns a **`list-of-sym`**

Also, if we give **`filter`** a **`list-of-sym`**, then it calls **`PRED`** with symbols only

The Contract of Filter

The reason **`filter`** works is that if we give it a **`list-of-sym`**, then it returns a **`list-of-sym`**

Also, if we give **`filter`** a **`list-of-sym`**, then it calls **`PRED`** with symbols only

A better contract:

```
filter :
((num -> bool) list-of-num
 -> list-of-num)
OR
((sym -> bool) list-of-sym
 -> list-of-sym)
```

The Contract of Filter

The reason **`filter`** works is that if we give it a **`list-of-sym`**, then it returns a **`list-of-sym`**

Also, if we give **`filter`** a **`list-of-sym`**, then it calls **`PRED`** with symbols only

A better contract:

```
filter :
((num -> bool) list-of-num
 -> list-of-num)
OR
((sym -> bool) list-of-sym
 -> list-of-sym)
```

But what about a list of **`image`**s, **`posn`**s, or **`snake`**s?

The True Contract of Filter

The real contract is

```
filter : ((X -> bool) list-of-X -> list-of-X)
```

where **`X`** stands for any type

- The caller of **`filter`** gets to pick a type for **`X`**
- All **`X`**s in the contract must be replaced with the same type

The True Contract of Filter

The real contract is

```
filter : ((X -> bool) list-of-X -> list-of-X)
```

where **`X`** stands for any type

- The caller of **`filter`** gets to pick a type for **`X`**
- All **`X`**s in the contract must be replaced with the same type

Data definitions need type variables, too:

```
; A list-of-X is either
;   - empty
;   - (cons X empty)
```

Using Filter

The **`filter`** function is so useful that is't built in

New solution for HW 4 that works in **Intermediate**:

```
(define (eat-apples l)
  (local [(define (not-apple? s)
            (not (symbol=? s 'apple)))]
    (filter not-apple? l)))
```

Looking for Other Built-In Functions

Recall **`inflate-by-4%`**:

```
; inflate-by-4% : list-of-num -> list-of-num
(define (inflate-by-4% l)
  (cond
    [(empty? l) empty]
    [else (cons (* (first l) 1.04)
                (inflate-by-4% (rest l)))]))
```

Is there a built-in function to help?

Looking for Other Built-In Functions

Recall **inflate-by-4%**:

```
; inflate-by-4% : list-of-num -> list-of-num
(define (inflate-by-4% l)
  (cond
    [(empty? l) empty]
    [else (cons (* (first l) 1.04)
                (inflate-by-4% (rest l)))]))
```

Is there a built-in function to help?

Yes: **map**

Using Map

```
(define (map CONV l)
  (cond
    [(empty? l) empty]
    [else (cons (CONV (first l))
                (map CONV (rest l)))]))


(define (inflate-by-4% l)
  (local [(define (inflate-one n)
            (* n 1.04))]
    (map inflate-one l)))

; negate-colors : list-of-col -> list-of-col
(define (negate-colors l)
  (map negate-color l))
```

The Contract for Map

```
(define (map CONV l)
  (cond
    [(empty? l) empty]
    [else (cons (CONV (first l))
                (map CONV (rest l)))]))
```

- The **l** argument must be a list of **X**
- The **CONV** argument must accept each **X**
- If **CONV** returns a new **X** each time, then the contract for **map** is

```
map : (X -> X) list-of-X -> list-of-X
```

Posns and Distances

Another function from HW 4:

```
; distances : list-of-posn -> list-of-num
(define (distances l)
  (cond
    [(empty? l) empty]
    [(cons? l) (cons (distance-to-0 (first l))
                     (distances (rest l)))]))
```

Posns and Distances

Another function from HW 4:

```
; distances : list-of-posn -> list-of-num
(define (distances l)
  (cond
    [(empty? l) empty]
    [(cons? l) (cons (distance-to-0 (first l))
                     (distances (rest l)))]))
```

The **`distances`** function looks just like **`map`**, except that **`distances-to-0`** is

`posn -> num`

not

`posn -> posn`

The True Contract of Map

Despite the contract mismatch, this works!

```
(define (distances l)
  (map distance-to-0 l))
```

The True Contract of Map

Despite the contract mismatch, this works!

```
(define (distances l)
  (map distance-to-0 l))
```

The true contract of **`map`** is

```
map : (X -> Y) list-of-X -> list-of-Y
```

The caller gets to pick both **`X`** and **`Y`** independently

More Uses of Map

```
; modernize : list-of-pipe -> list-of-pipe
(define (modernize l)
  ; replaces 4 lines:
  (map modern-pipe l))

; modern-pipe : pipe -> pipe
...

; rob-train : list-of-car -> list-of-car
(define (rob-train l)
  ; replaces 4 lines:
  (map rob-car l))

; rob-car : car -> car
...
```

Folding a List

How about **`sum`**?

```
sum : list-of-num -> num
```

Doesn’t return a list, so neither **`filter`** nor **`map`** help

Folding a List

How about **sum**?

sum : list-of-num -> num

Doesn't return a list, so neither **filter** nor **map** help

But recall **combine-nums**...

```
; combine-nums : list-of-num num
; (num num -> num) -> num
(define (combine-nums l base-n COMB)
  (cond
    [(empty? l) base-n]
    [(cons? l)
     (COMB
      (first l)
      (combine-nums (rest l) base-n COMB))]))
```

The Foldr Function

```
; foldr : (X Y -> Y) Y list-of-X -> Y
(define (foldr COMB base l)
  (cond
    [(empty? l) base]
    [(cons? l)
     (COMB (first l)
           (foldr COMB base (rest l)))]))
```

The Foldr Function

```
; foldr : (X Y -> Y) Y list-of-X -> Y
(define (foldr COMB base l)
  (cond
    [(empty? l) base]
    [(cons? l)
     (COMB (first l)
           (foldr COMB base (rest l)))]))
```

The **sum** and **product** functions become trivial:

```
(define (sum l) (foldr + 0 l))
(define (product l) (foldr * 1 l))
```

The Foldr Function

```
; foldr : (X Y -> Y) Y list-of-X -> Y
(define (foldr COMB base l)
  (cond
    [(empty? l) base]
    [(cons? l)
     (COMB (first l)
           (foldr COMB base (rest l)))]))
```

Useful for HW 5:

```
; total-blue : list-of-col -> num
(define (total-blue l)
  (local [(define (add-blue c n)
            (+ (color-blue c) n))]
    (foldr add-blue 0 l)))
```

The Foldr Function

```
; foldr : (X Y -> Y) Y list-of-X -> Y
(define (foldr COMB base l)
  (cond
    [(empty? l) base]
    [(cons? l)
     (COMB (first l)
           (foldr COMB base (rest l)))]))
```

In fact,

```
(define (map f l)
  (local [(define (comb i r)
             (cons (f i) r))]
    (foldr comb empty l)))
```

The Foldr Function

```
; foldr : (X Y -> Y) Y list-of-X -> Y
(define (foldr COMB base l)
  (cond
    [(empty? l) base]
    [(cons? l)
     (COMB (first l)
           (foldr COMB base (rest l)))]))
```

Yes, **`filter`** too:

```
(define (filter f l)
  (local [(define (check i r)
            (cond
              [(f i) (cons i r)]
              [else r]))]
    (foldr check empty l)))
```

The Source of Foldr

How can `foldr` be so powerful?

The Source of Foldr

Template:

```
(define (func-for-loX l)
  (cond
    [(empty? l) ...]
    [(cons? l) ... (first l)
     ... (func-for-loX (rest l)) ...]))
```

Fold:

```
(define (foldr COMB base l)
  (cond
    [(empty? l) base]
    [(cons? l)
     (COMB (first l)
           (foldr COMB base (rest l)))]))
```

Other Built-In List Functions

More specializations of `foldr`:

```
ormap : (X -> bool) list-of-X -> bool

andmap : (X -> bool) list-of-X -> bool
```

Examples:

```
; got-milk? : list-of-sym -> bool
(define (got-milk? l)
  (local [(define (is-milk? s)
             (symbol=? s 'milk))]
    (ormap is-milk? s)))

; all-passed? : list-of-grade -> bool
(define (all-passed? l)
  (andmap passing-grade? l))
```

What about Non-Lists?

Since it's based on the template, the concept of fold is general

```
; fold-ftn : (sym num sym Z Z -> Z) Z ftn -> Z
(define (fold-ftn COMB base ftn)
  (cond
    [(empty? ftn) base]
    [(child? ftn)
     (COMB (child-name ftn) (child-date ftn) (child-eyes ftn)
           (fold-ftn COMB BASE (child-father ftn))
           (fold-ftn COMB BASE (child-mother ftn)))]))

(define (count-persons ftn)
  (local [(define (add name date color c-f c-m)
            (+ 1 c-f c-m))]
    (fold-ftn add 0 ftn)))

(define (in-family? who ftn)
  (local [(define (here? name date color in-f? in-m?)
            (or (symbol=? name who) in-f? in-m?))]
    (fold-ftn here? false ftn)))
```