A Belief-Based Account of Decision under Uncertainty

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Outline

- Problem Definition
- Decision under Uncertainty (classical Theory)
- Two-Stage Model
- Probability Judgment and Support Theory
- The Case Studies and Discussion

Decision under Uncertainty

- Judgment of probability

- Decision under Risk

Two studies

- 1995 professional basketball playoffs
- Movement of economic indicators in a simulated economy
- Results ....
  - Consistent with the belief-based account
  - Violated the partition inequality (implied by classical theory of decision under uncertainty)

Decision Making

- *Decision*: Depends on the strength of people's *belief* an event happens.

- Question: How to measure these beliefs ?!

Decision under Uncertainty (classical Theory)

- "… derives beliefs about the likelihood of uncertain events from people's <u>choices</u> between prospects whose consequences are contingent on these events."

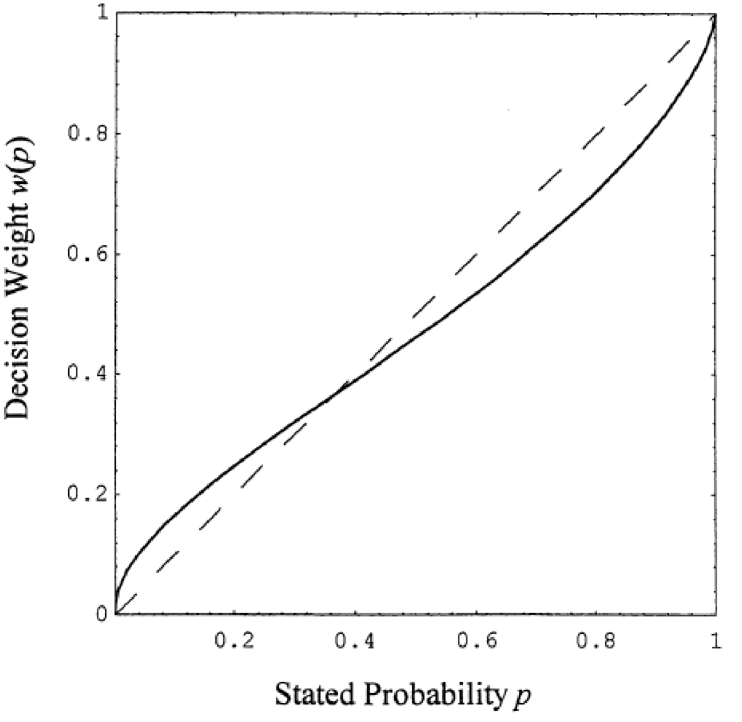
- Simultaneous measurement of utility and subjective probability

Challenges

- From psychological perspective:
  1) Belief precedes preference
  2) Probability Judgment
  3) The assumption of the derivation of belief from preference
- Belief based approach uses probability judgment to predict decisions under uncertainty

Background

- Risky prospects – known probabilities
  - Decision under risk
  - Non-linear weighting function



Background

- Real world decisions – uncertain prospects
- Extension to the domain of uncertainty

Cumulative Prospect Theory

- Assumes that an event has more impact on choice when:
  - *Possibility Effect:* It turns an impossibility into a possibility
  - *Certainty Effect:* It turns a possibility into a certainty than when it merely makes a possibility more or less likely.

Bounded Subadditivity

Bounded Subadditivity

- Tested on both risky and uncertain prospects.
- Data satisfied bounded subadditivity for both risk and uncertainty.
- Departure from expected utility theory

Two-Stage Model

- Decision makers:

  1) Assess the probability (P) of an uncertain event (A)
  2) Then, transform this value using the risky weighting function (w)

Two-Stage Model Terminology

- Simple prospect:
  - (x, A): Pay \$x, if the target event (A) obtains and nothing otherwise.
- Overall value of a prospect (V):
  - $V(x, A) = v(x)W(A) = v(x)w[P(A)]$
    - P(A): judged probability of A
    - v: value function for monetary gains
    - w: risky weighting function

Probability Judgment

- People's intuitive probability judgments are often inconsistent with the laws of chance.

- Support Theory: Probabilities are attached to *description of events* (called hypothesis) rather than the *events*.

Support Theory

- Hypothesis, A, has a nonnegative support value, s(A).

- Judged probability P(A, B):
  - Hypothesis A rather than B holds.
  - Interpreted as the support of the focal hypothesis, A, relative to the alternative hypothesis, B.

$$P(A, B) = \frac{s(A)}{s(A) + s(B)}$$

Support Theory

- The judged probability of the union of disjoint events is generally smaller than the sum of judged probabilities of these events.

$$s(A) \leq s(A_1 \vee A_2) \leq s(A_1) + s(A_2)$$

Unpacking principle

Support Theory

- Binary Complementarity:
  - $P(A, B) + P(B, A) = 1$
- Subaditivity:
  - For finer partitioning (i.e., more than 2 events), the judged probability is *less than or equal to* the sum of judged probabilities of its disjoint components.

$$s(A) \leq s(A_1) + s(A_2)$$

Implications

- Contrast between two-stage modern and expected utility theory with risk aversion:
  - The effect of partitioning

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- Contrast between two-stage modern and expected utility theory with risk aversion:
  - The effect of partitioning
  - The classical model follows *partition inequality*:

- C(x, A): $C(x, A_1) + \cdots + C(x, A_n) \leq C(x, A)$, pays $x if A occurs, and nothing otherwise.
- Doesn’t necessary hold considering the two-stage model.

Two-Stage Model

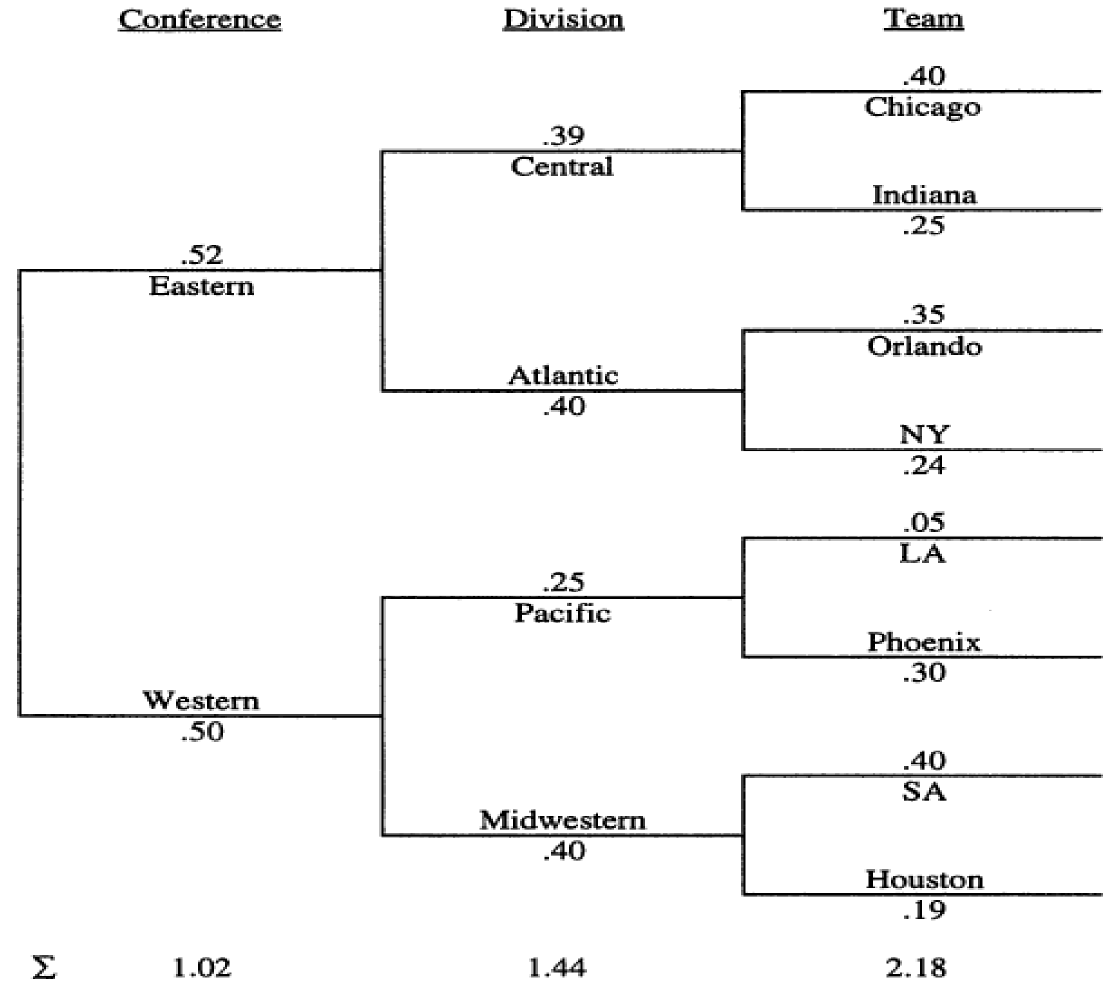
- Predict the certainty equivalent of an uncertain prospect, C(x, A) from two independent responses:
  - The judged probability of the target even, P(A)
  - The certainty equivalent of the risky prospect, C(x, P(A))

Study 1

- Four tasks:
  1) Estimating subjects' certainty equivalents (C) for risky prospects.
     - Random draw of a single poker ship from an urn
  2) Estimating subjects' certainty equivalents (C) for uncertain prospect.
     - offering reward if a particular team, division, or conference would win the 1995 NBA.
  3) An independent test of risk aversion
  4) Estimate the probability of target events.

Result of Study 1

Figure 2 Median Judged Probabilities for All Target Events in Study 1



Result of Study 1

- Fit of the models to the data
- Unpacking principle VS. monotonicity

Study 2

- More simulated environment :
  - Subjects have identical information
  - Compare the judged probabilities vs. actual probabilities

Result of Study 2

- Binary partitioning:
  - Judged probabilities (nearly) satisfy binary complementarity
  - Certainty equivalents satisfy the partition inequality
- Finer partitioning:
  - Subadditivity of judged probabilities
  - Reversal of the partition inequality for certainty equivalents

Discussion

- The event spaces under study has some structure (hierarchical, product, …)
- Subaditivity of judged probability is a major cause of violations of the partition inequality
- Generalization of the two-stage model for source preference.
- Particular description of events on which outcomes depends may affect a person’s willingness to act (unpacking principle)