Surface Completion of an Irregular Boundary Curve Using a Concentric Mapping

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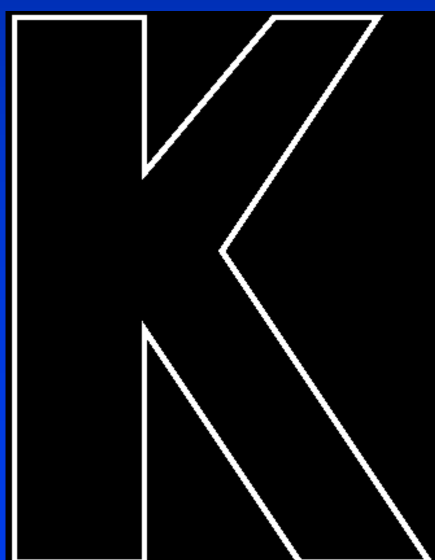
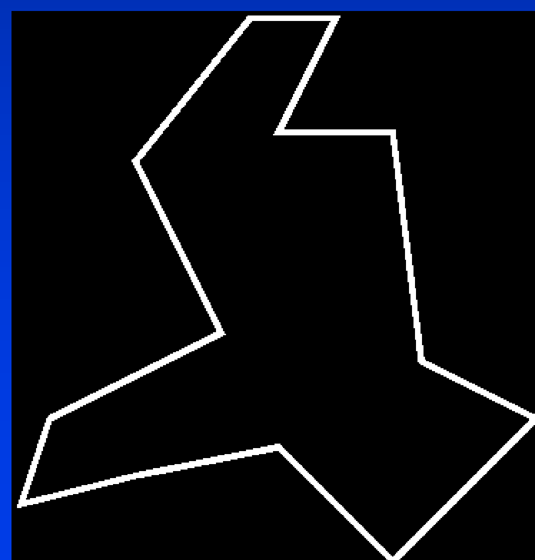
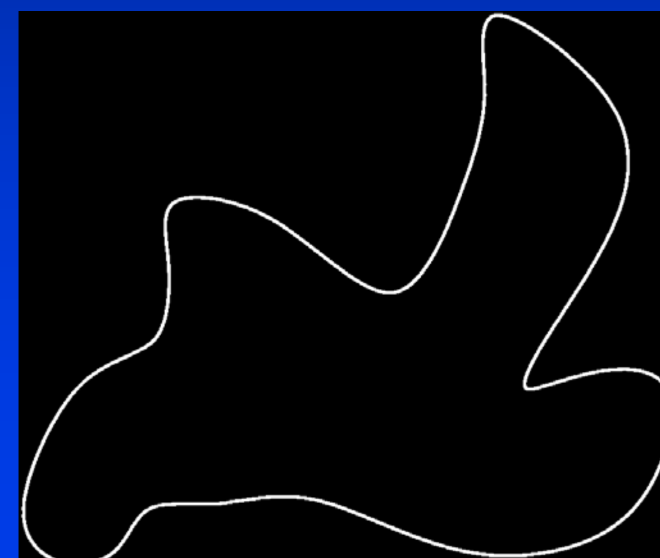
Problem Statement

Problem: How to create a parametric representation of a region bounded by a closed planar curve?

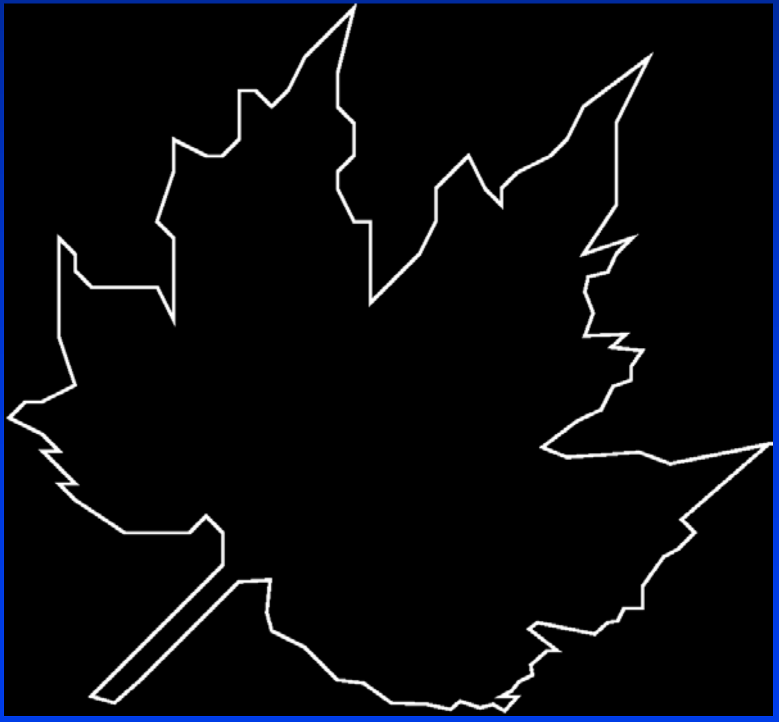


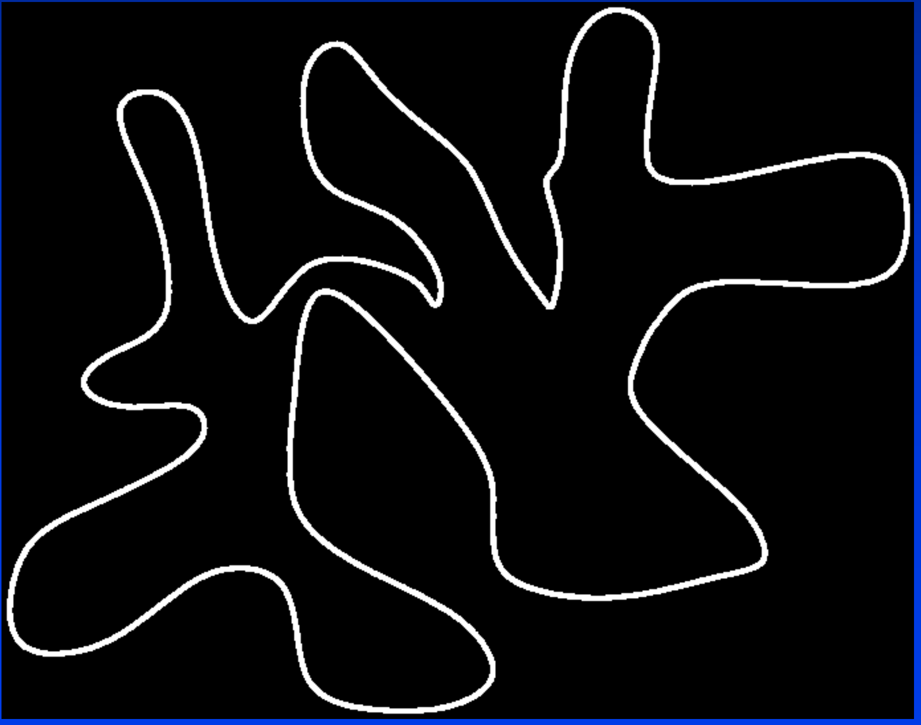




More Complex Examples

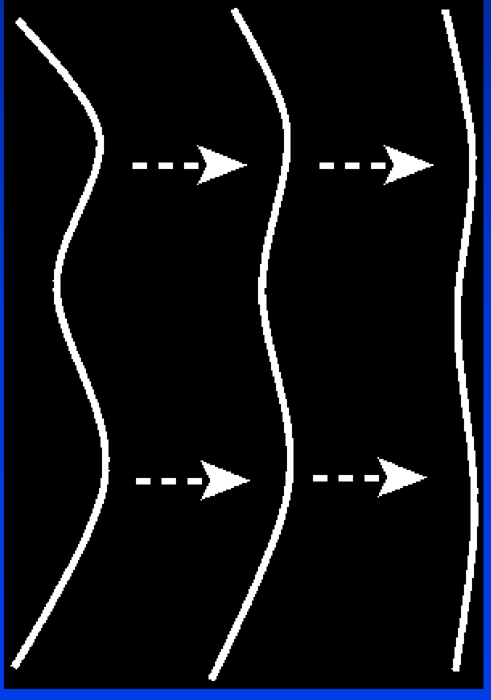
However, things can get pretty ugly….

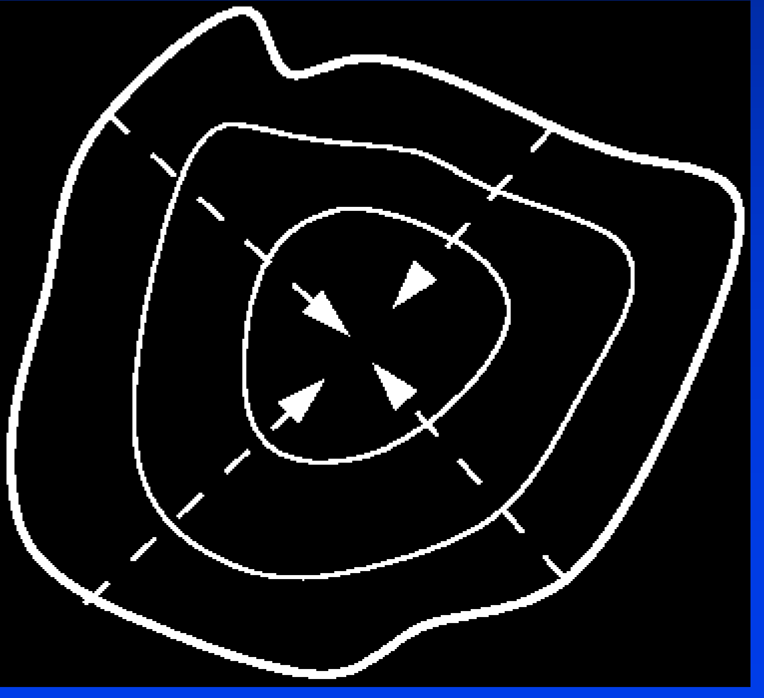




Core Idea

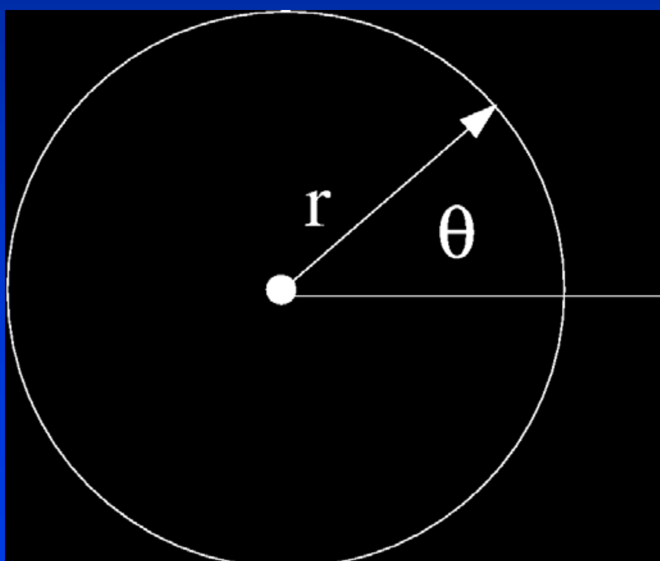
New parameterization based on the concept of concentric moving curves.

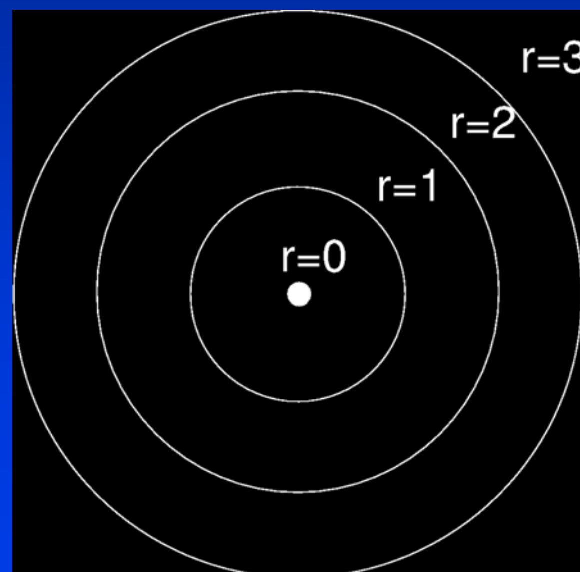


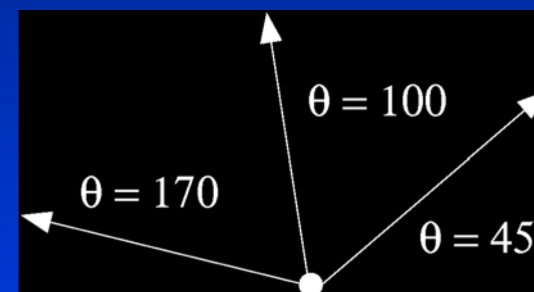


Classical Circle Parameterization

Our parameterization is inspired by the classical polar parameterization of a circle.

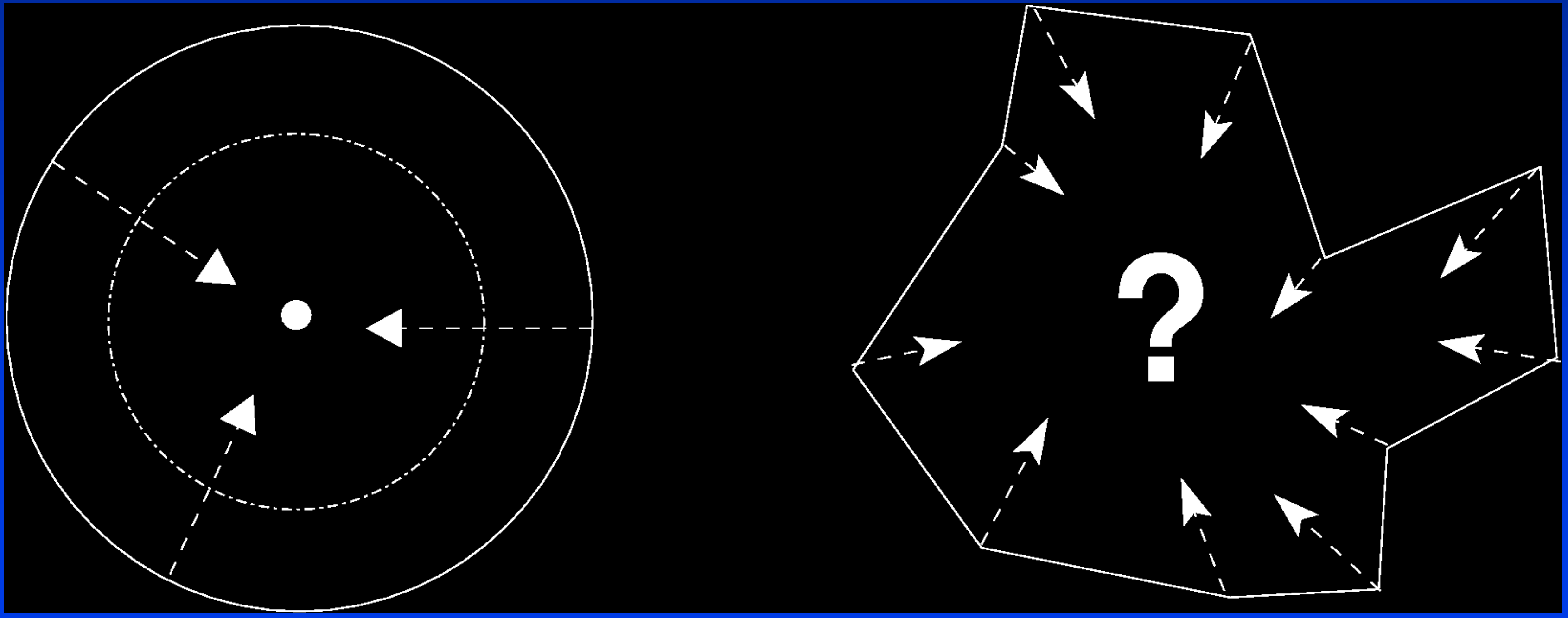






Generalization

A generalization to irregular boundaries requires a generalization of the notion of a center point.

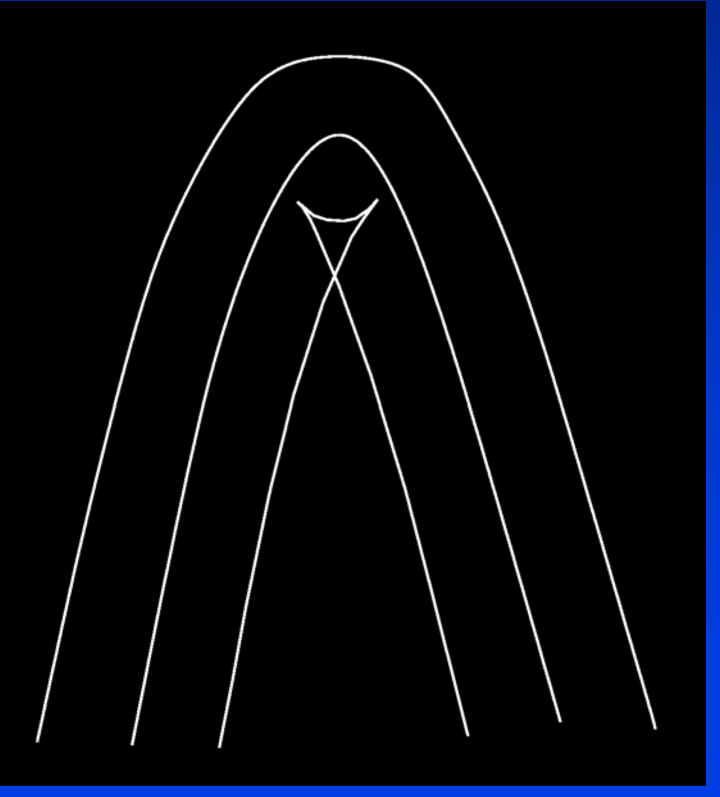


Offset Curves

Consider the sequence of offset curves as isoparametric curves in r.

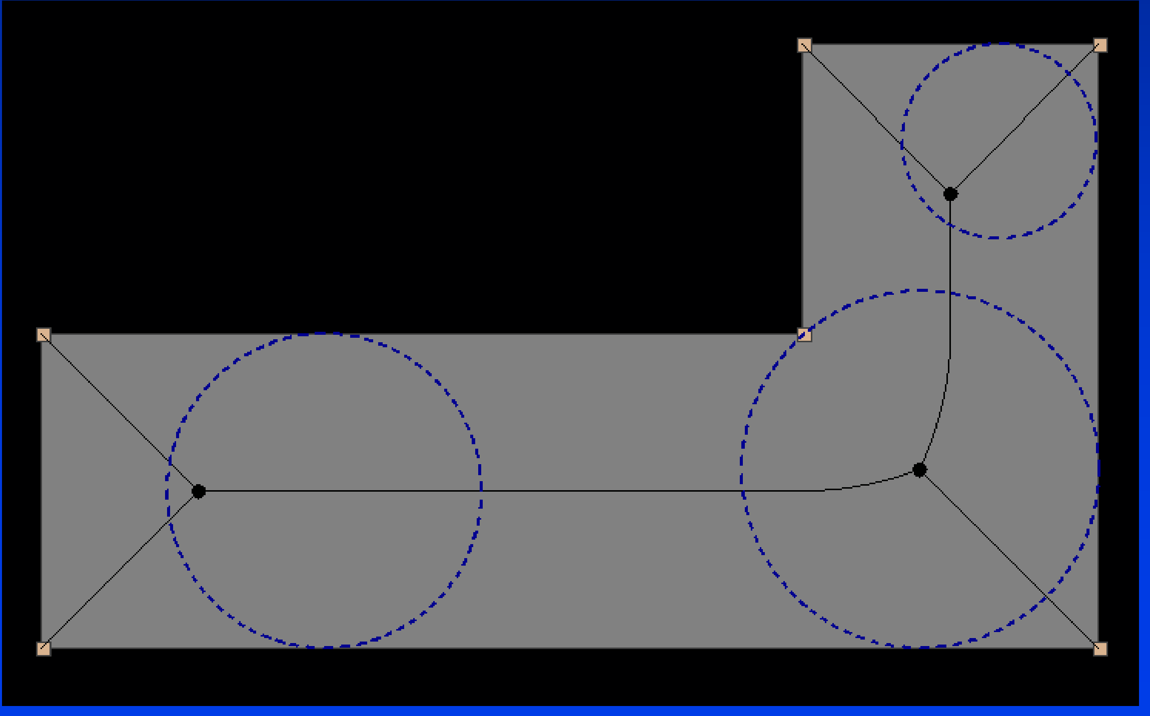
Requires that the crossings be clipped.

The associated parameter intervals are mapped to a point.



Connection to Medial Axis

Medial Axis: locus of the centers of all inscribed circles that touch the boundary at 2 or more points.



More Medial Axis

In fact, the Medial Axis Transform contains more information. Each point on the medial axis has an associated radius which give the size of the maximal disc centered at that point.

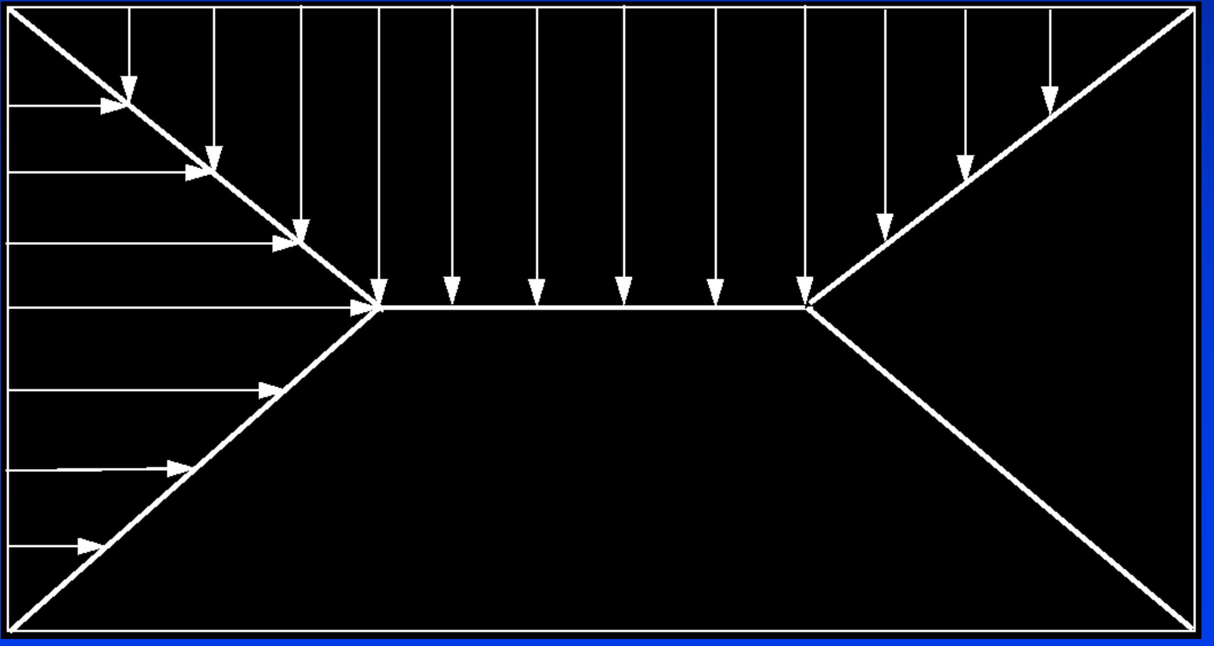
The union of the discs gives the interior of the boundary.

Medial Axis Parameterization

This immediately implies a parameterization of the interior of the polygon:

$(1-s) * P + s * MA(P)$

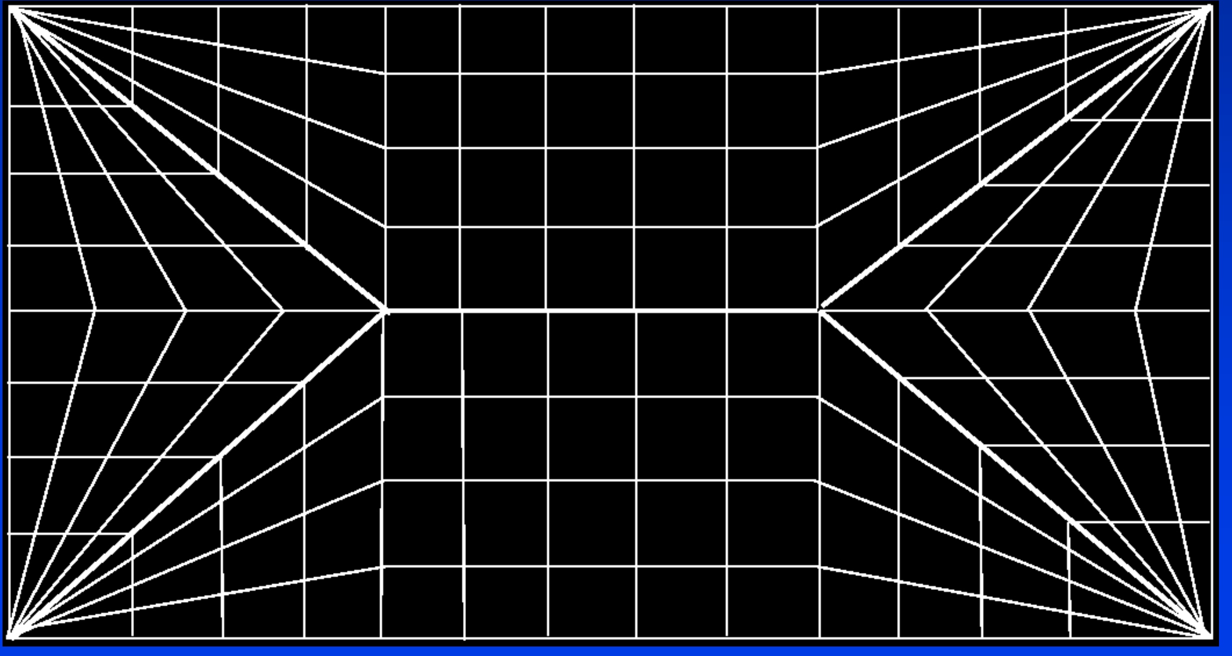
Homotopy.



Flaws in the Medial Axis Mapping

This mapping is generally not to be preferred:

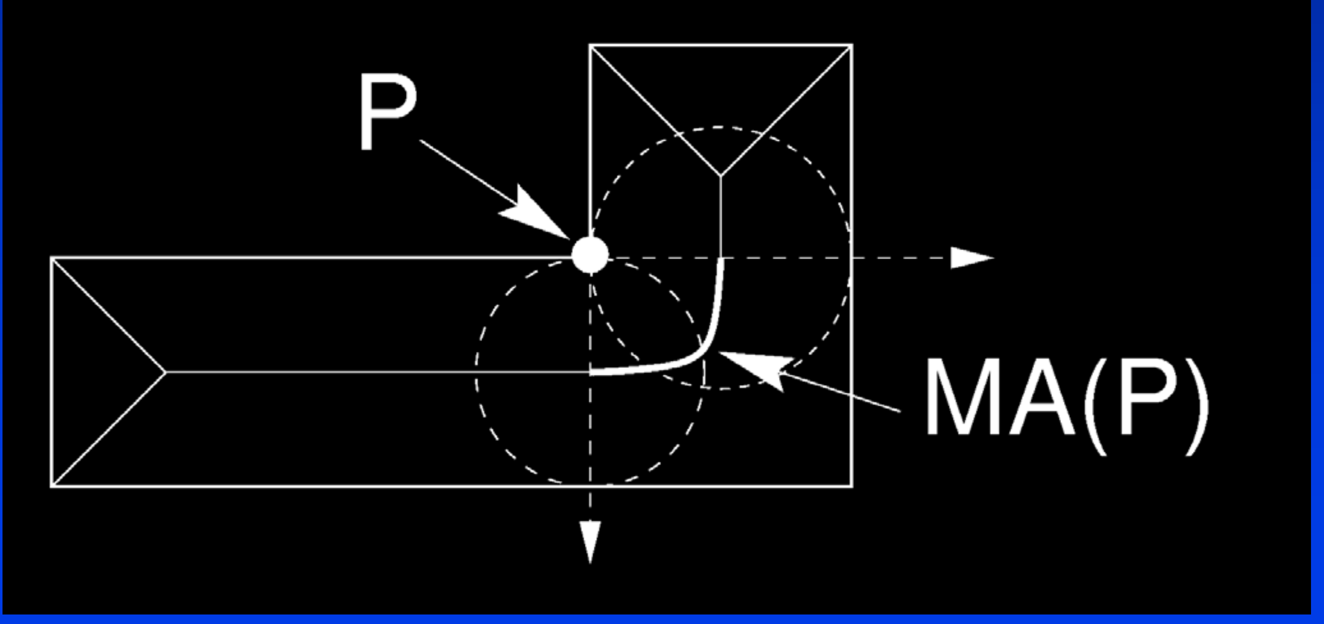
- Some points are fixed, some move a LOT.



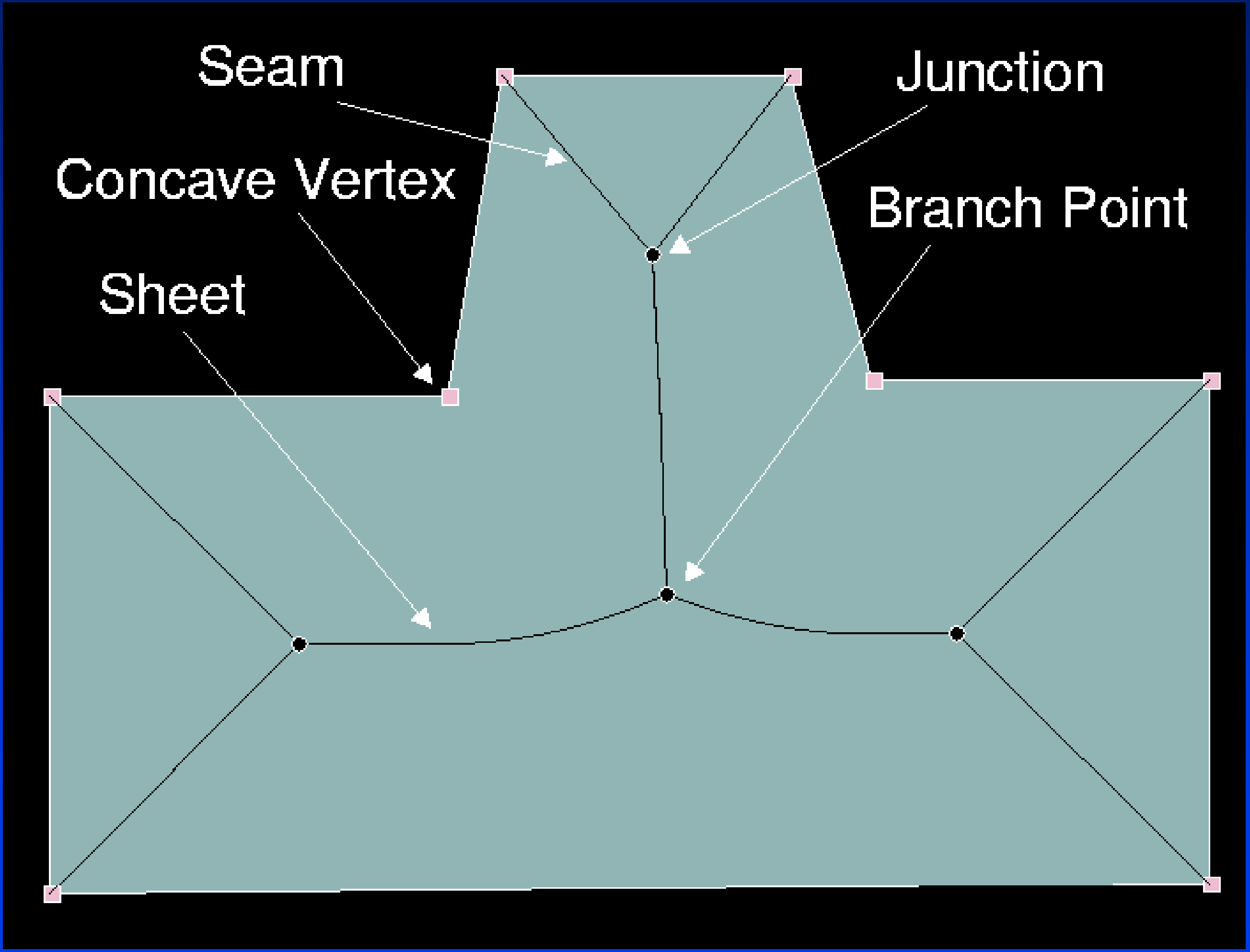
Flaws in the Medial Axis Mapping

This mapping is generally not to be preferred:

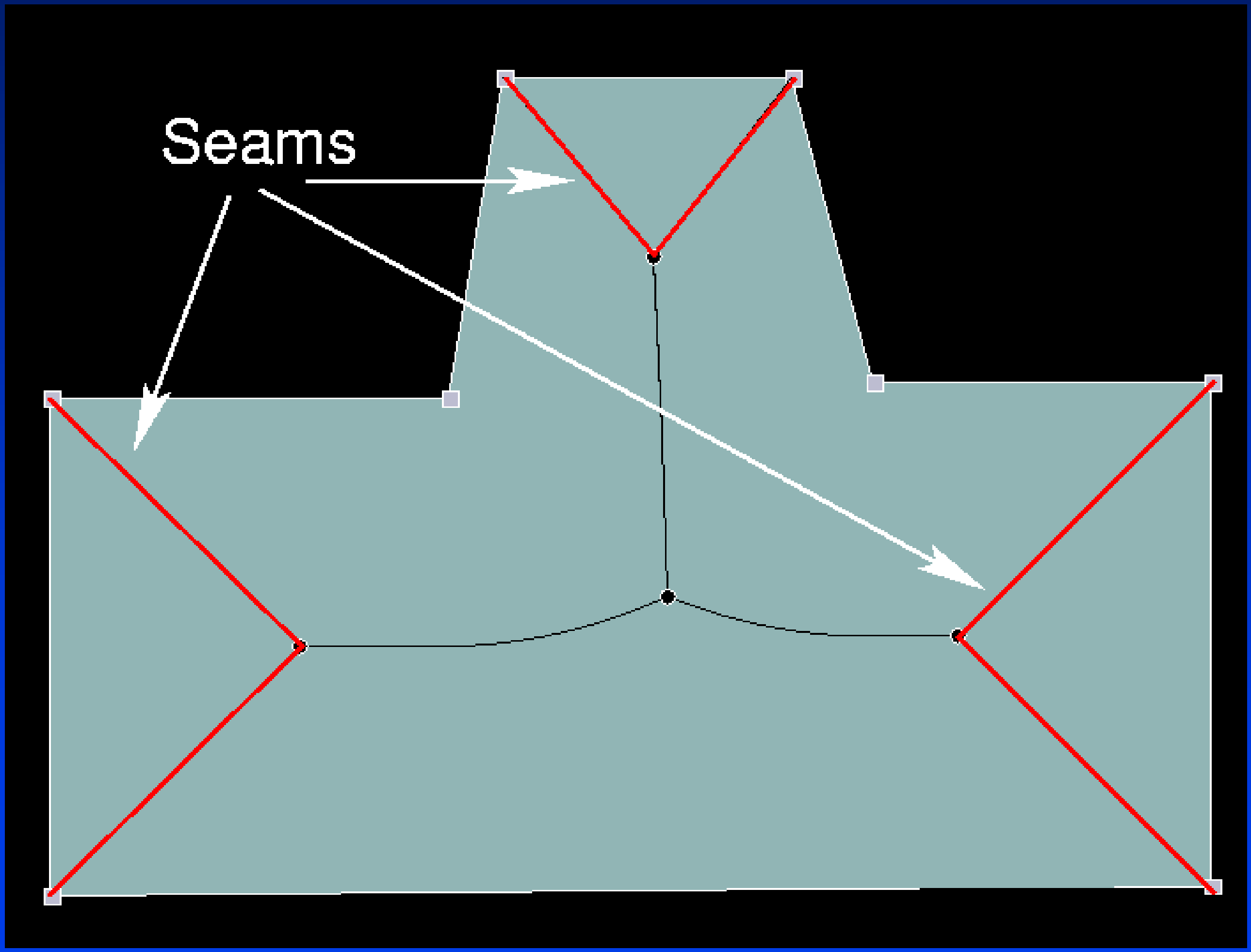
- Not necessarily a function.



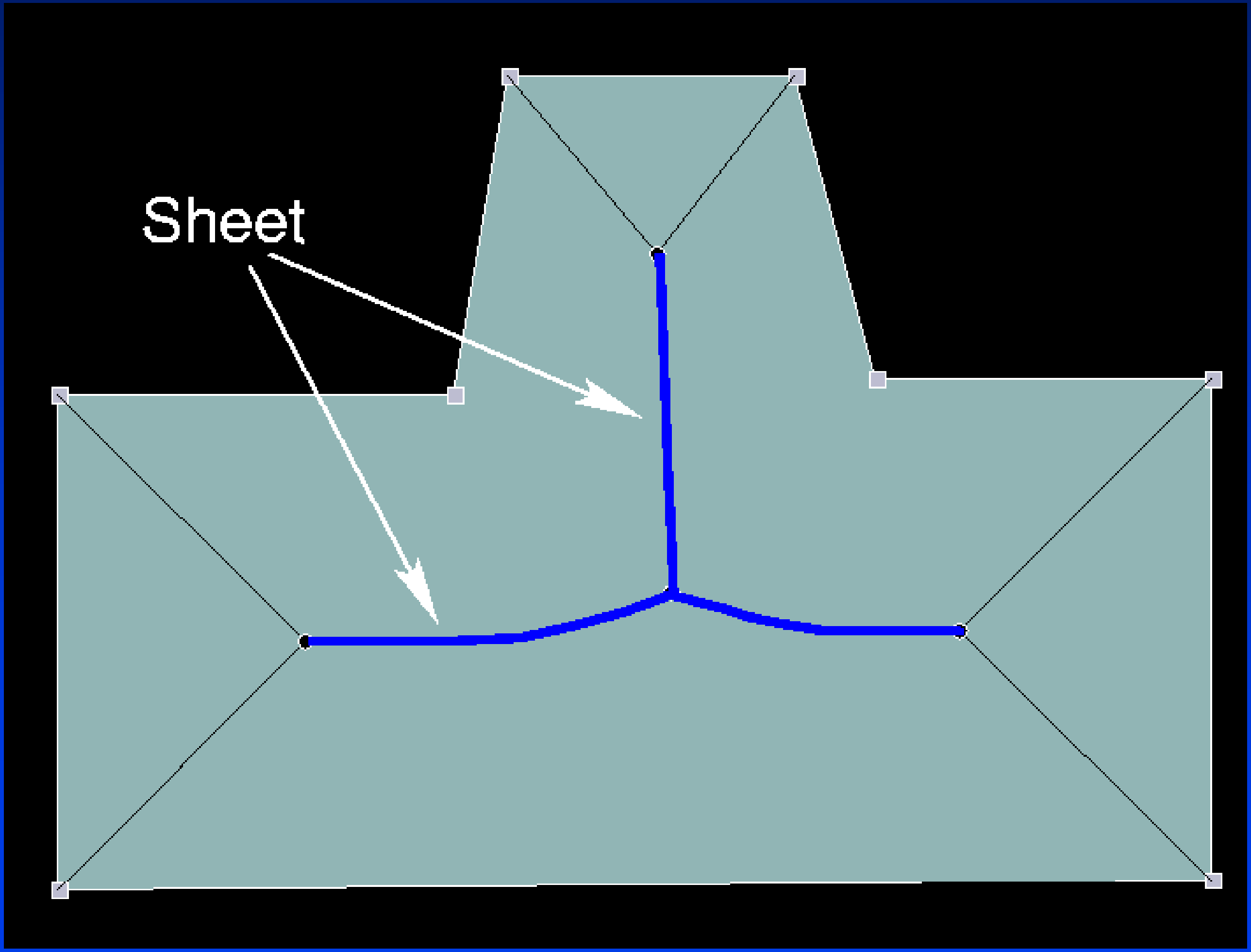
Medial Axis Nomenclature



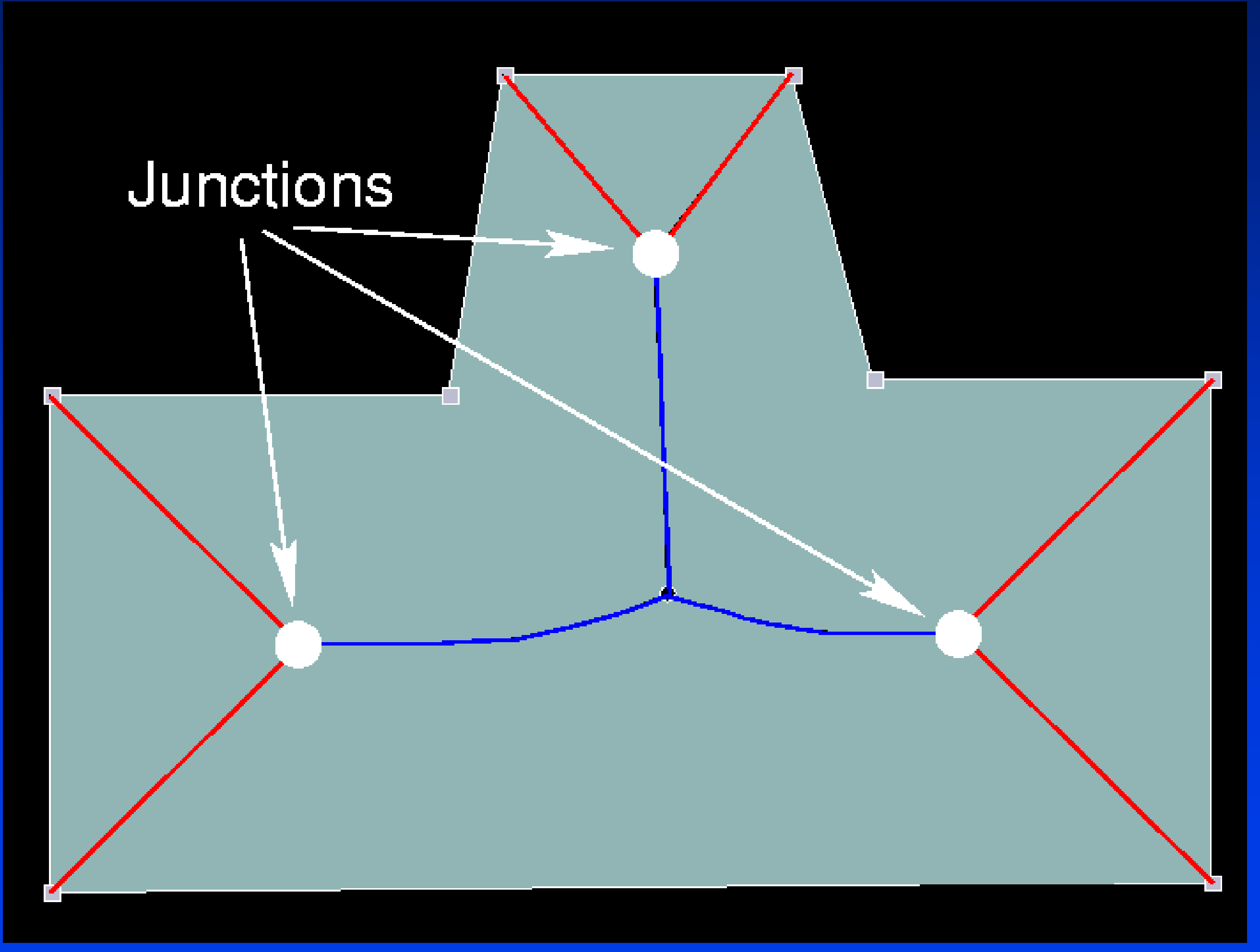
Medial Axis Nomenclature



Medial Axis Nomenclature



Medial Axis Nomenclature



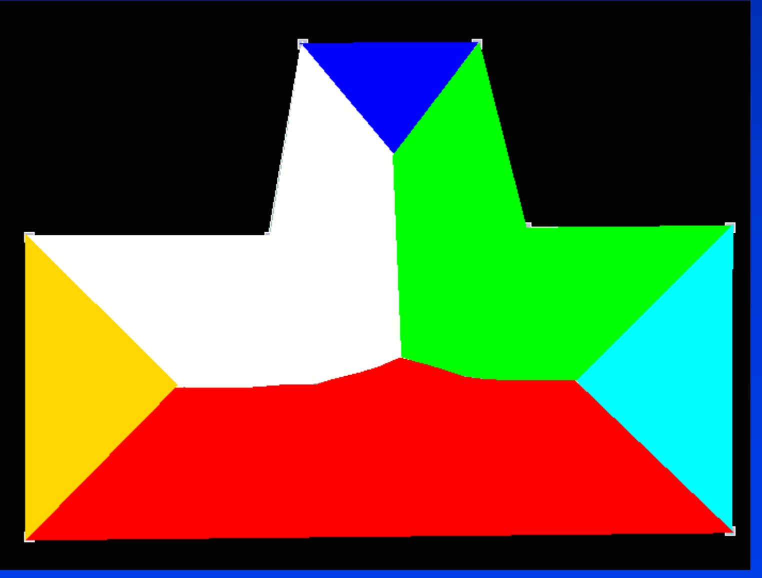
Polygon Concentric Mapping

The seams divide the interior into regions.

Consider the mapping M of the edges of the polygon onto the sheet using the seams as guides.

$$(1-s)*E + s*M(E)$$

Assign parameter values to the boundary.

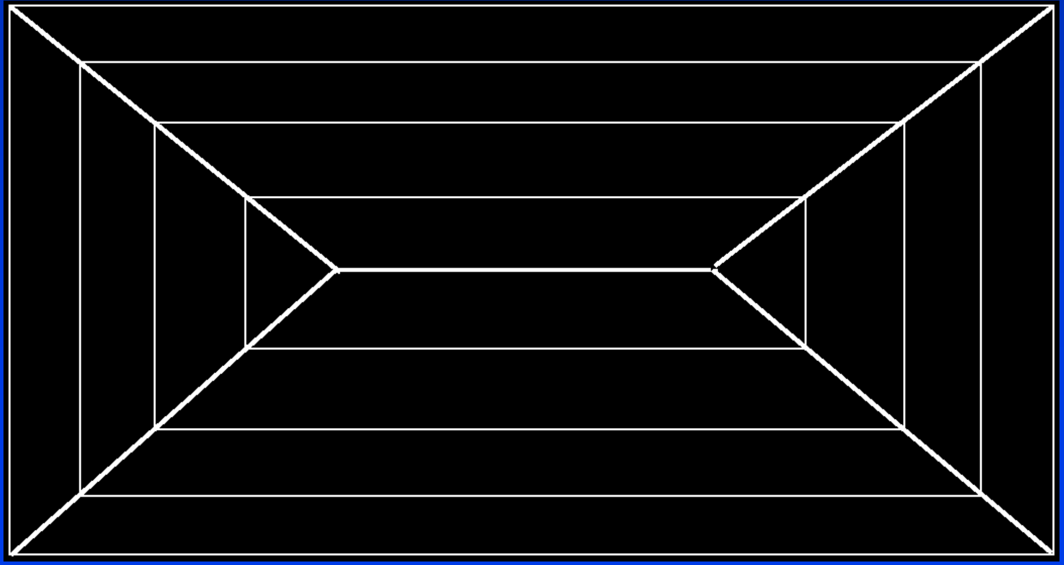


Positive Attributes of the Mapping

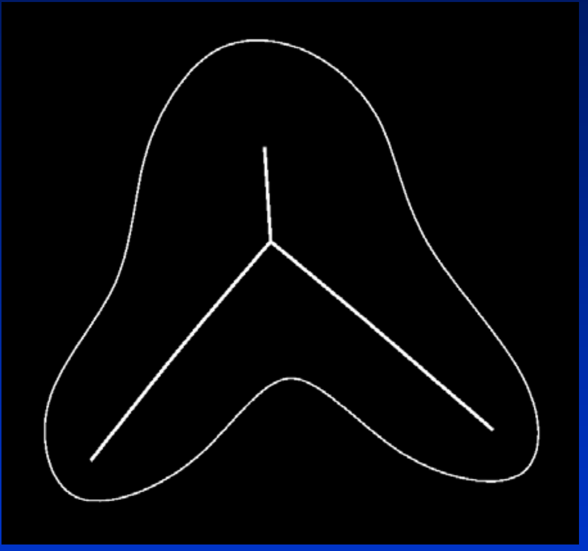
Parameterization matches across seams.

Simple Coons patch formulation.

Non-overlapping parameter domains.



Generalization to parametric curves



- Somewhat difficult to calculate.
- No handles makes contraction difficult.
- Instead, we choose to generalize the preceding mapping by approximation.

Leveraging B-Spline Properties

- Defined by a control polygon.
- Convex hull property.
- Variation diminishing property.

Mapping the control polygon to the medial axis implies a mapping of the curve to a curve defined on the medial axis.

Quality of the approximate mapping

- Control polygon is an approximation to the curve.
- Control polygon converges to the curve quadratically under uniform refinement.
- Therefore, limit behavior is identical to the mapping used for polygons.
- Obviously the better the approximation to the curve, the better the parameterization.

B-Spline Concentric Mapping

Assumptions:

- Control polygon is a reasonable approximation to curve.
- We can reasonably approximate the medial axis sheet with piecewise linear segments.
- Closed curve, with periodic end conditions.

B-Spline Concentric Mapping

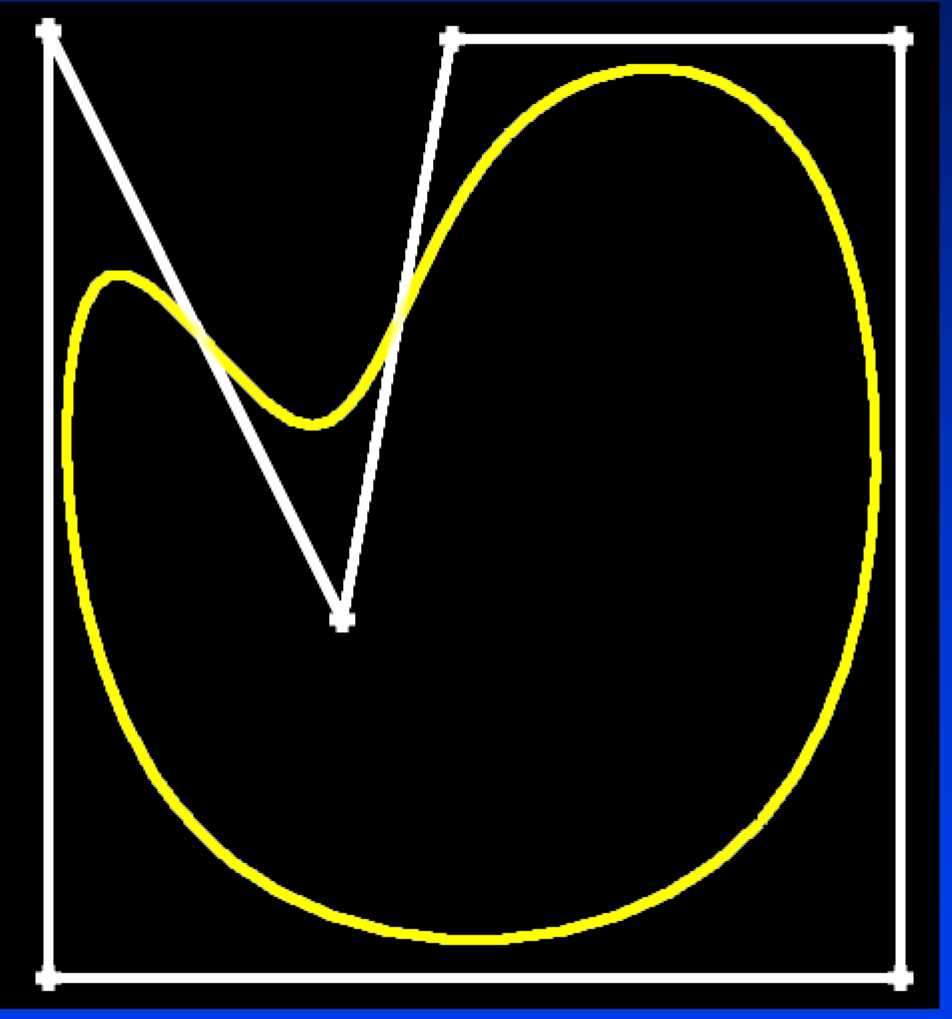
Overview:

- Approximate the medial axis sheet with linear segments.
- Subdivide the regions so that all regions are either 3 or 4 sided, mapping boundary segments to either medial points or medial segments.
- Use refinement to make the boundary and medial curves compatible, and to ensure correspondence between vertices on the boundary and medial vertices.
- Compute a sweep surface from the boundary and medial curves.

Example Curve

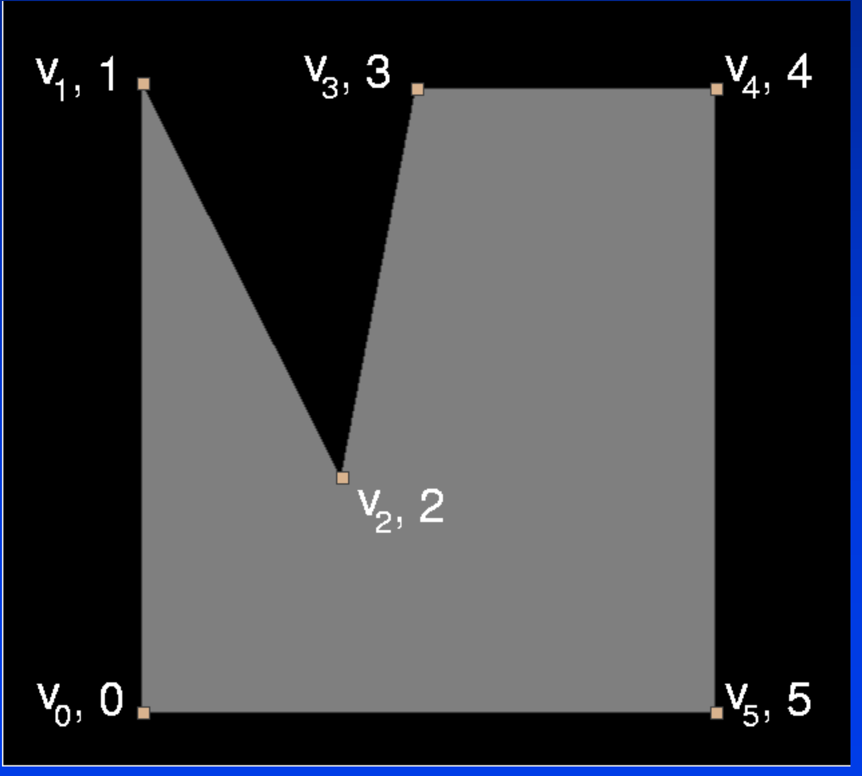
Degree 3

Uniform knot vector



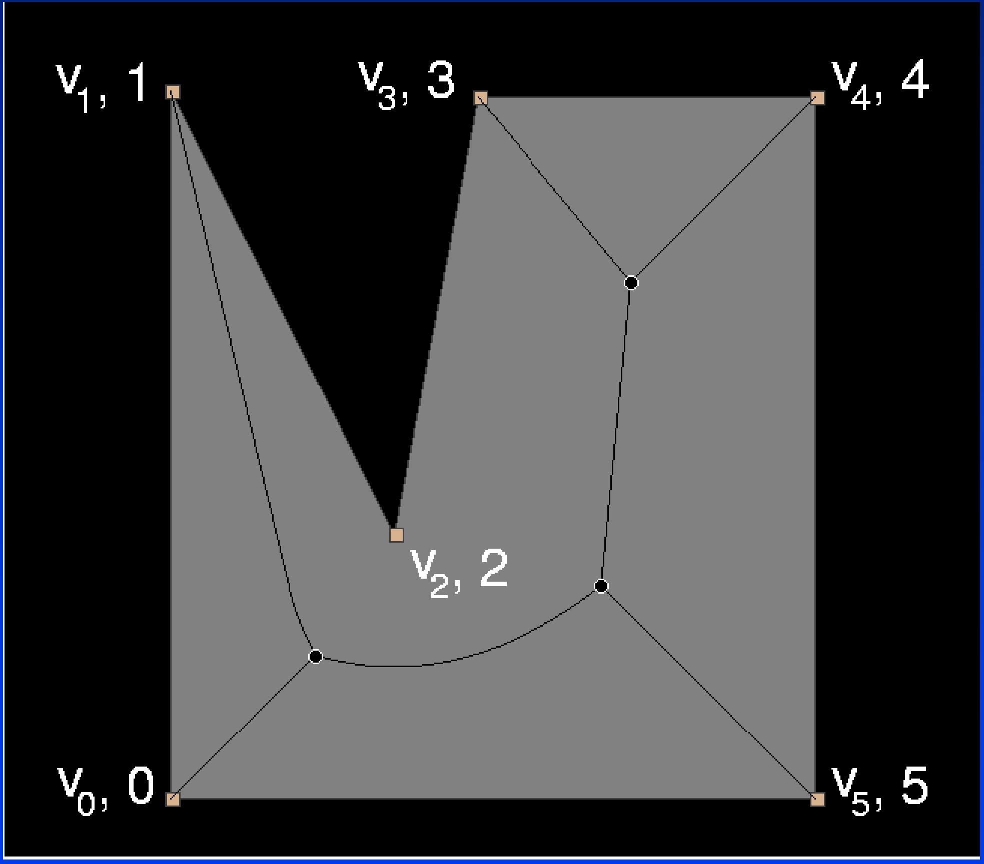
Concentric Mapping Algorithm

- 1) Imbue the control polygon with a parameterization using the nodal values of the spline space.



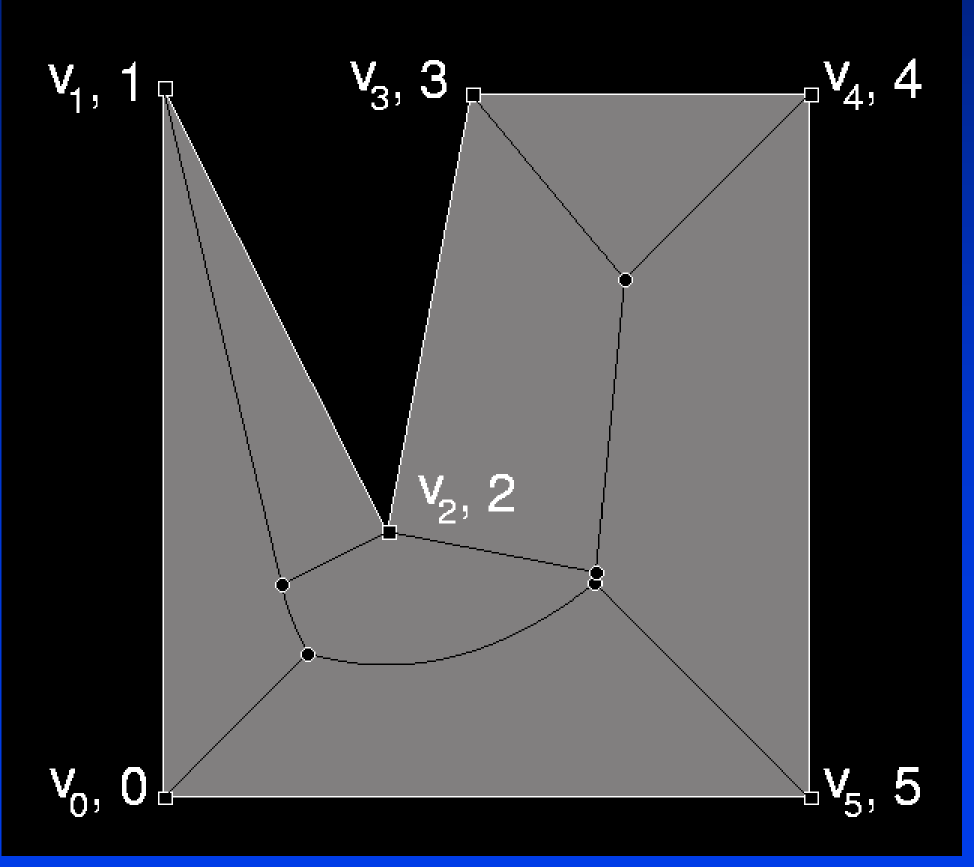
Concentric Mapping Algorithm

- 2) Calculate the medial axis of the control polygon.



Concentric Mapping Algorithm

- 3) For each concave vertex, insert a medial vertex.



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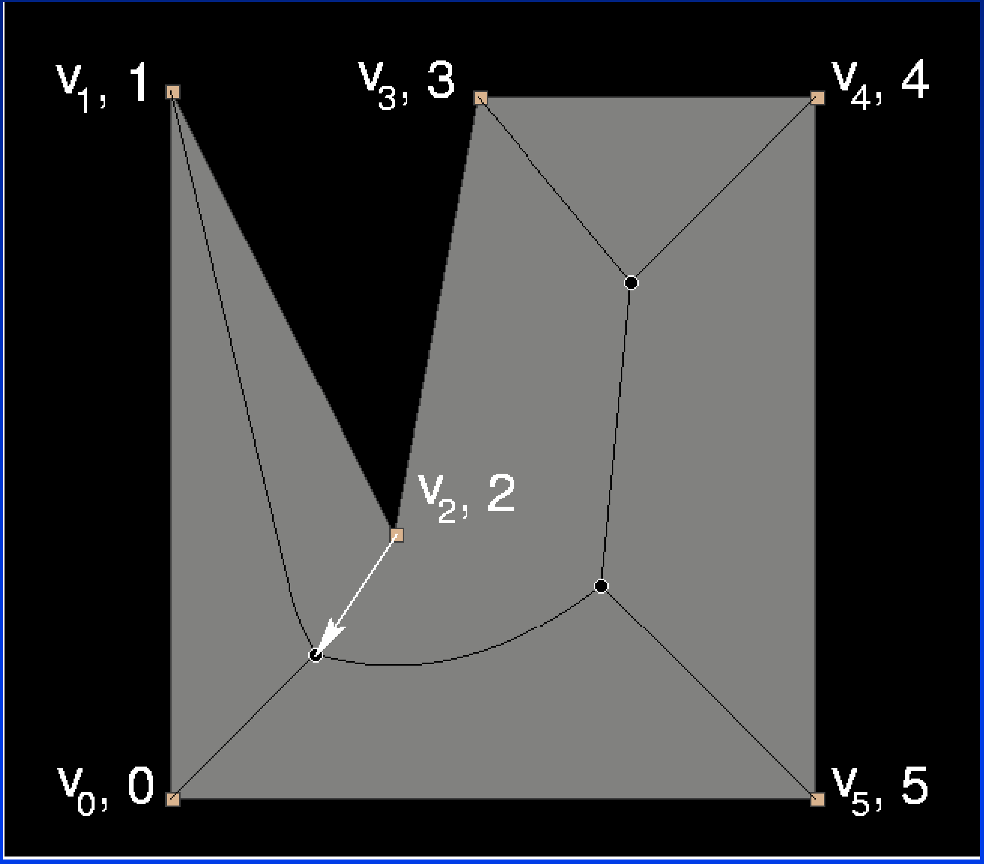
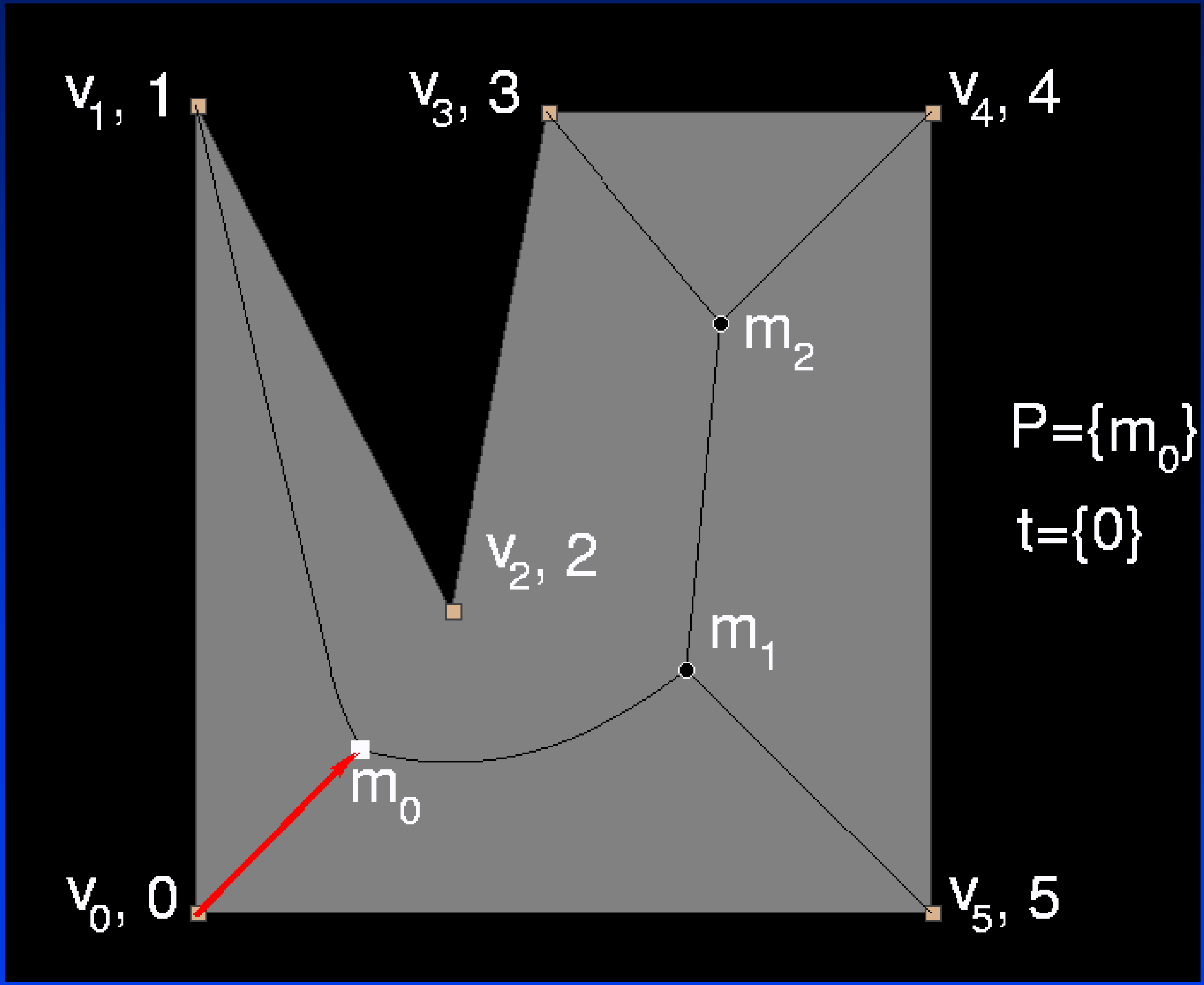


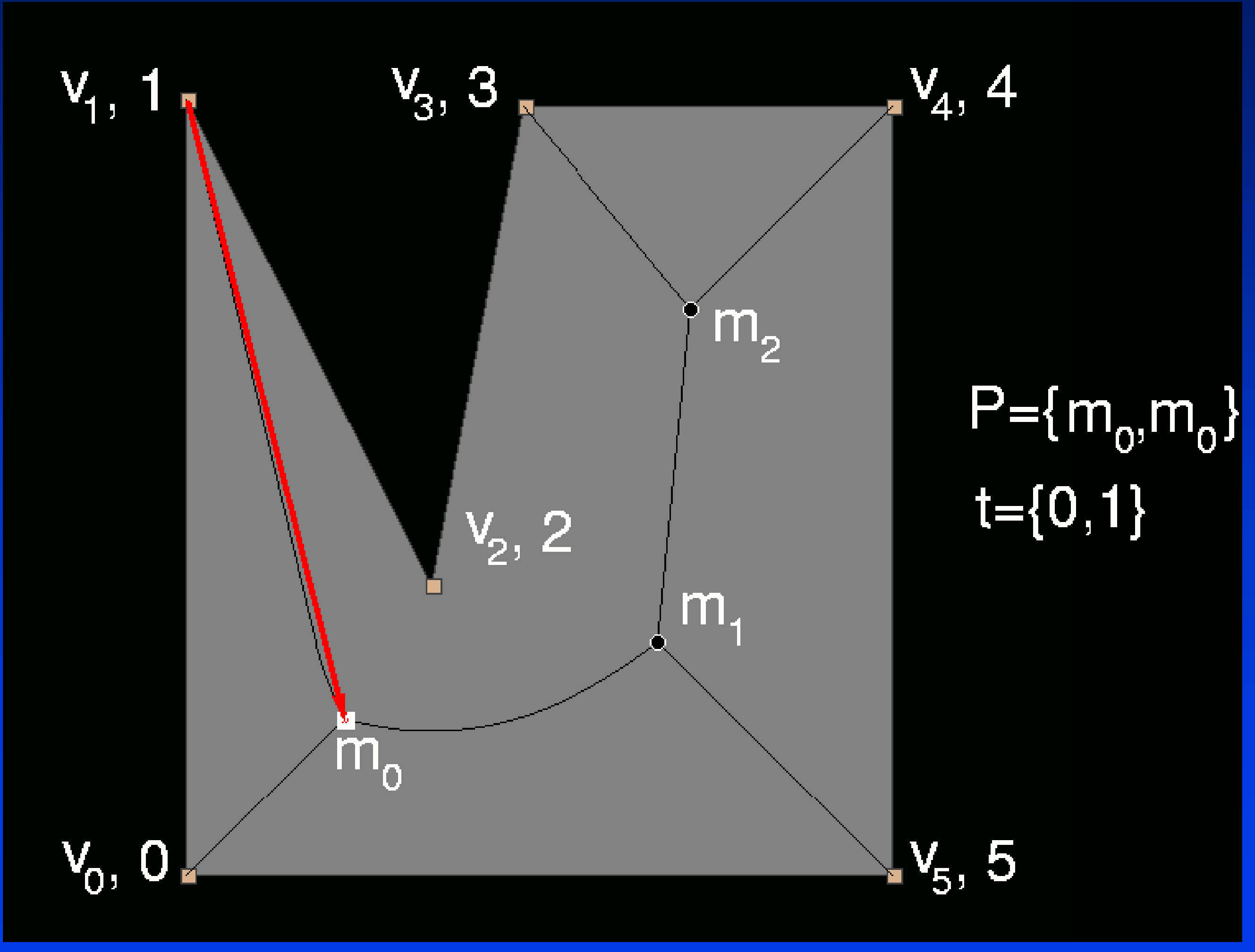
Concentric Mapping Algorithm

- 4) Form the medial curve control polygon: For each point on the control polygon, find the closest medial vertex along the seam, and add it to the medial control polygon. Add the associated nodal value to the knot vector.

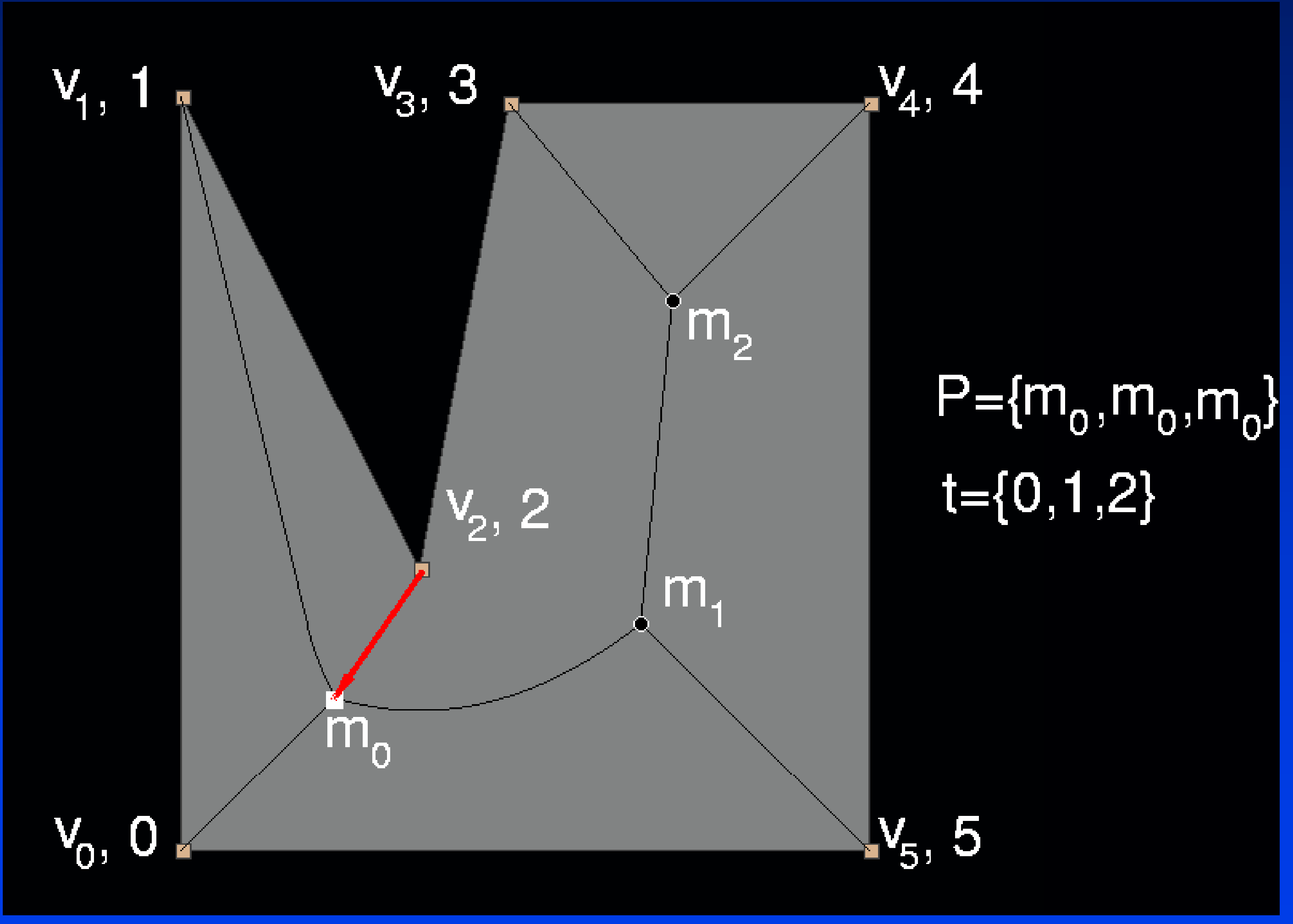
Concentric Mapping Algorithm



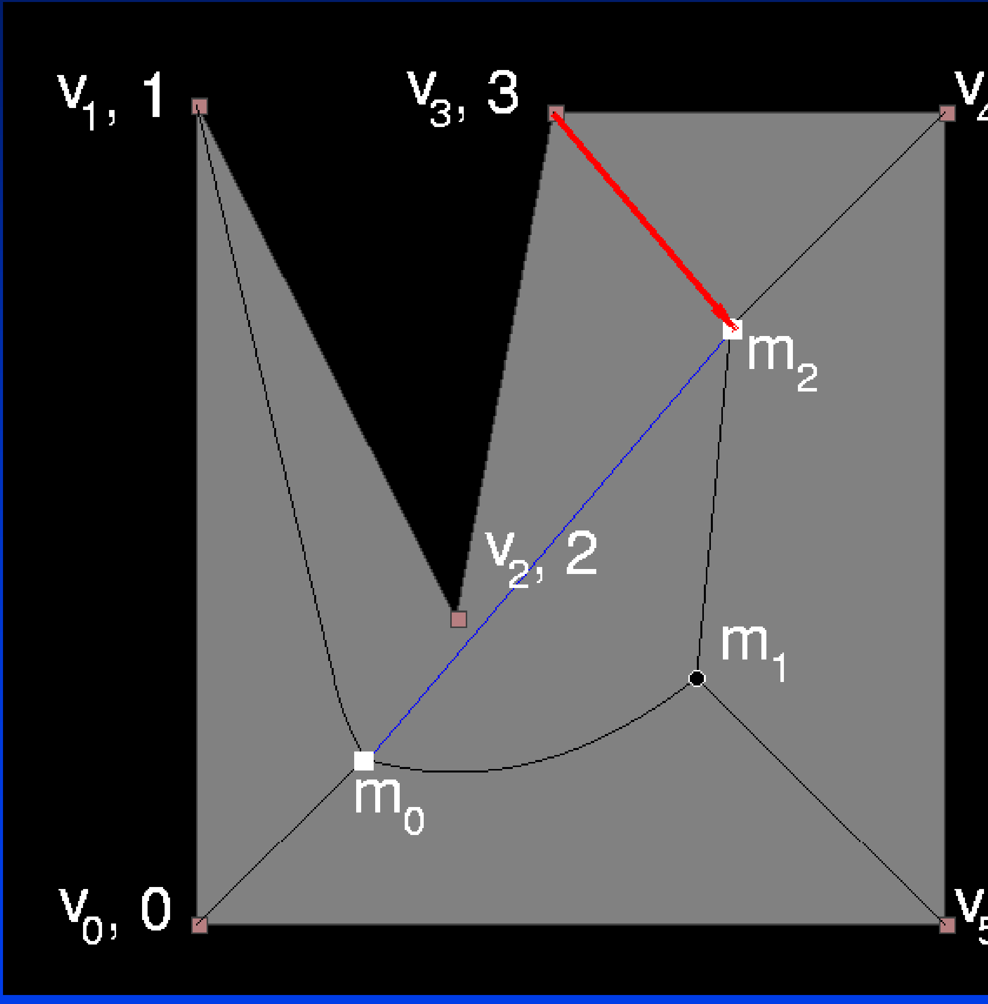
Concentric Mapping Algorithm



Concentric Mapping Algorithm



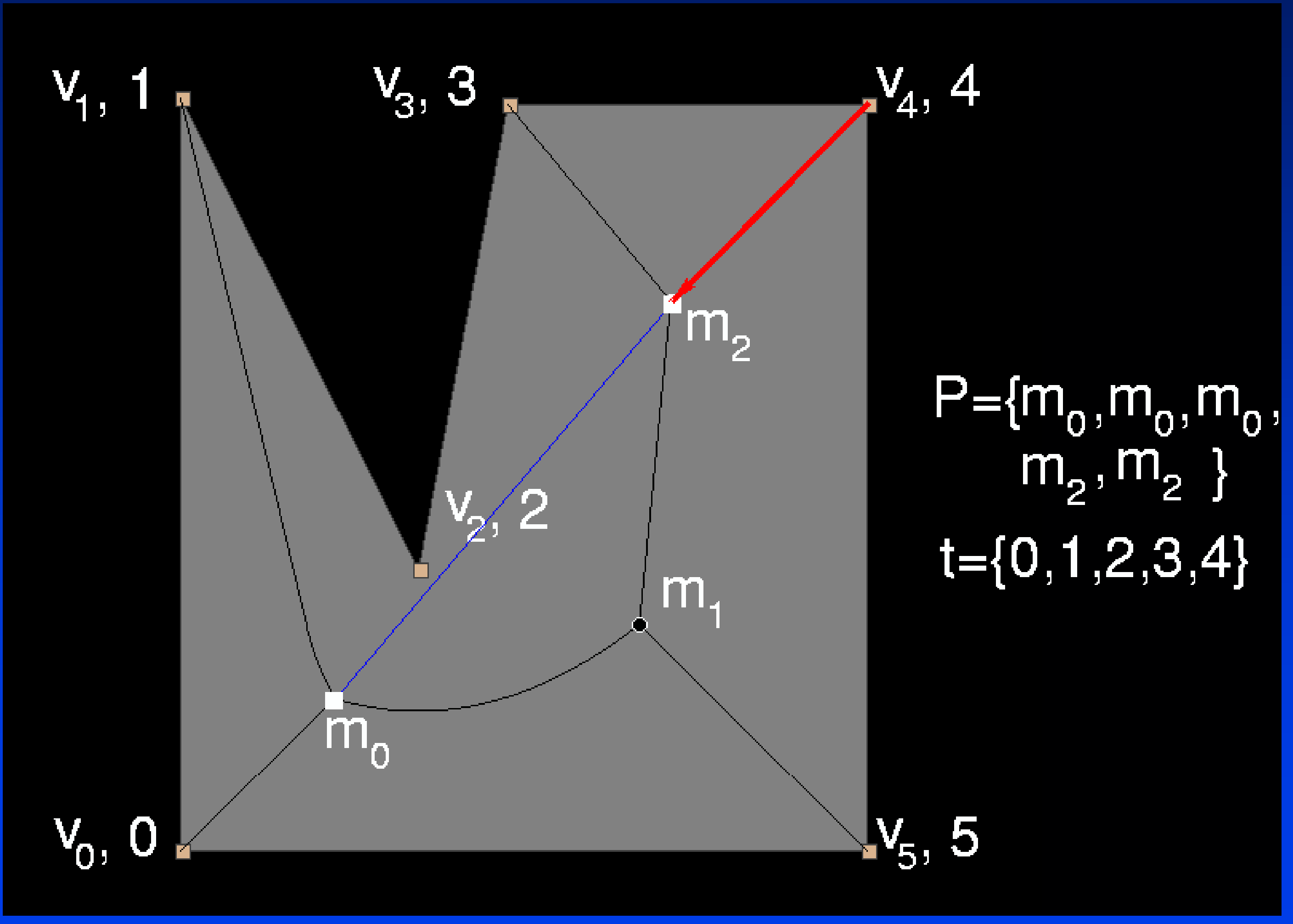
Concentric Mapping Algorithm



$P=\{m_0, m_0, m_0, m_2\}$

$t=\{0,1,2,3\}$

Concentric Mapping Algorithm



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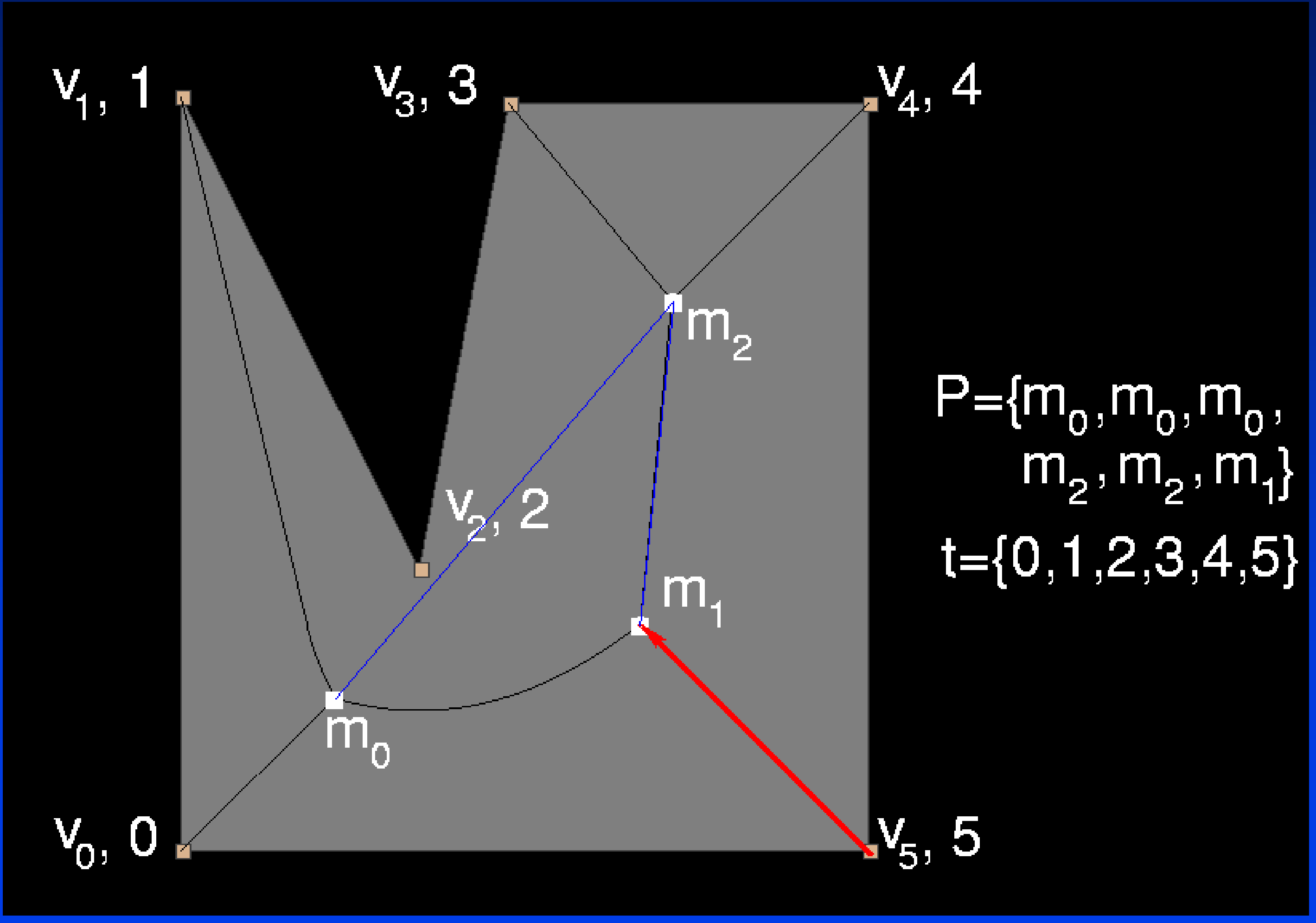
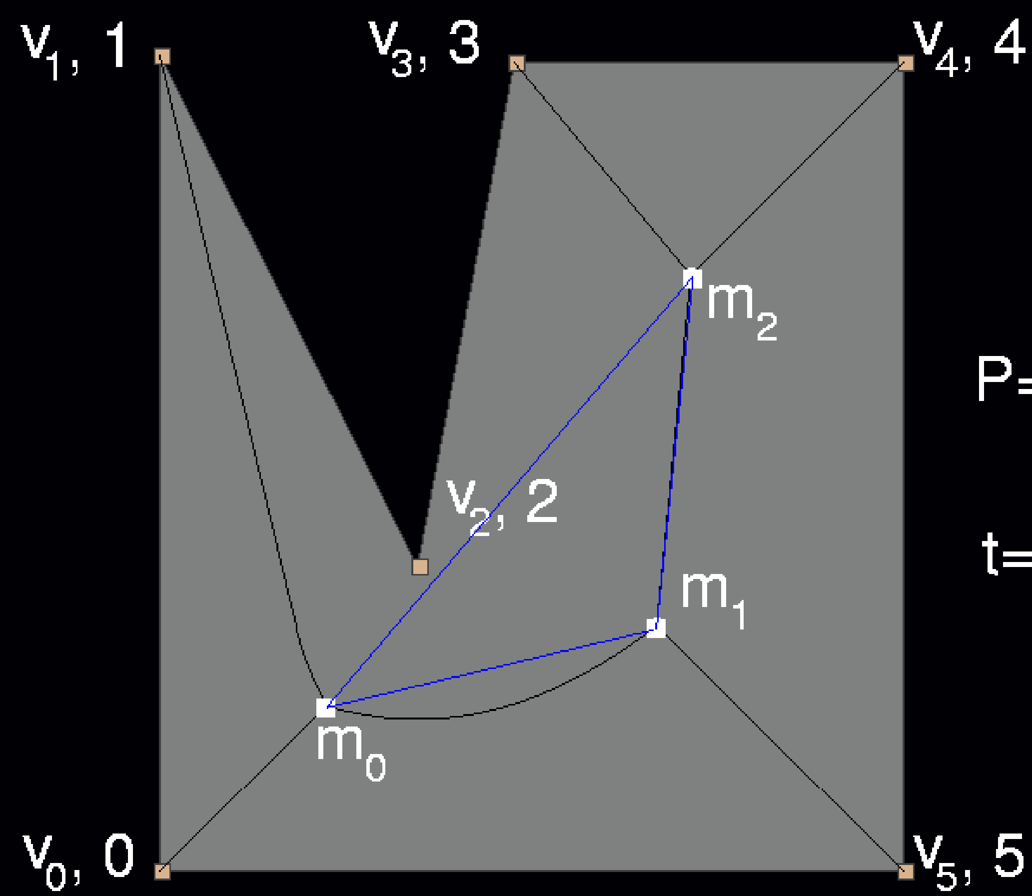


Concentric Mapping Algorithm



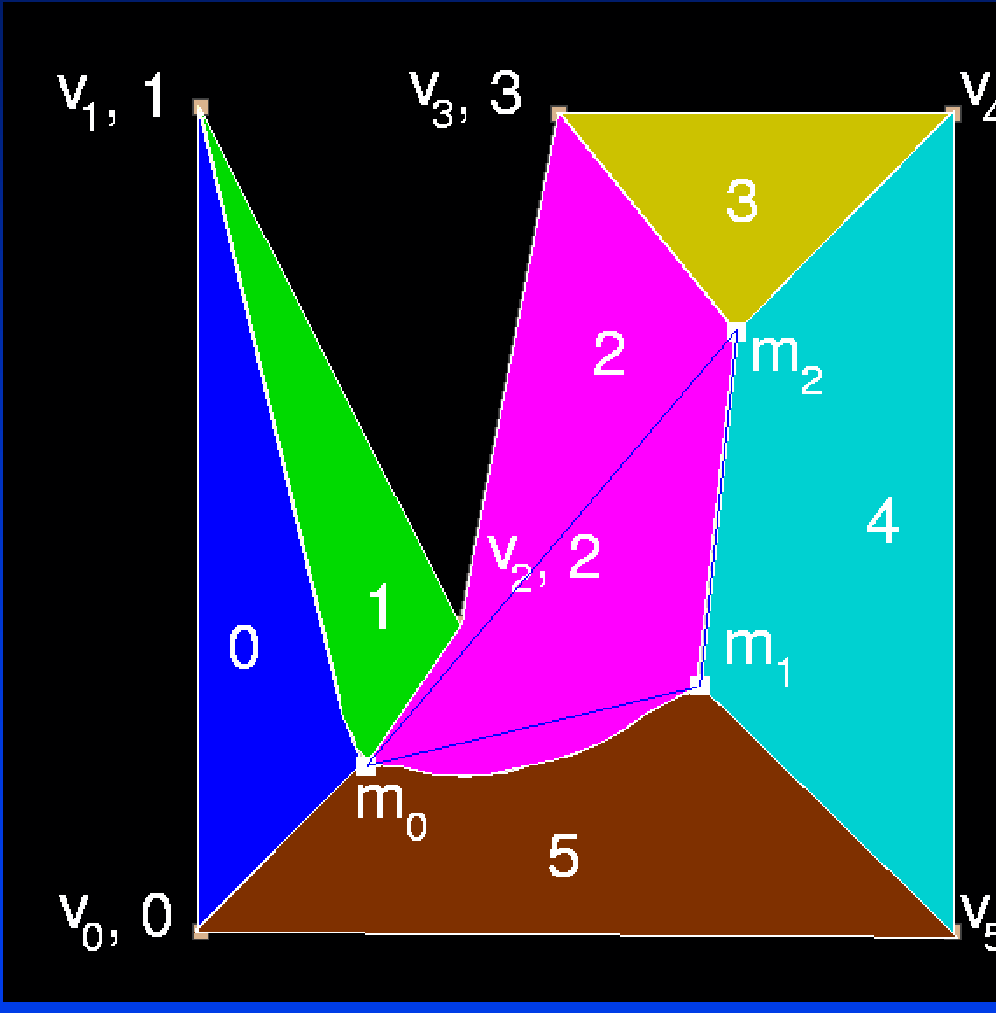
$P = \{m_0, m_0, m_0, m_2, m_2, m_1\}$

$t = \{0, 1, 2, 3, 4, 5, 6\}$

Concentric Mapping Algorithm

- 5) For each region, if there are internal medial vertices, insert them into the medial control polygon, and associate a knot value using the interpolated values of the endpoint knot values.

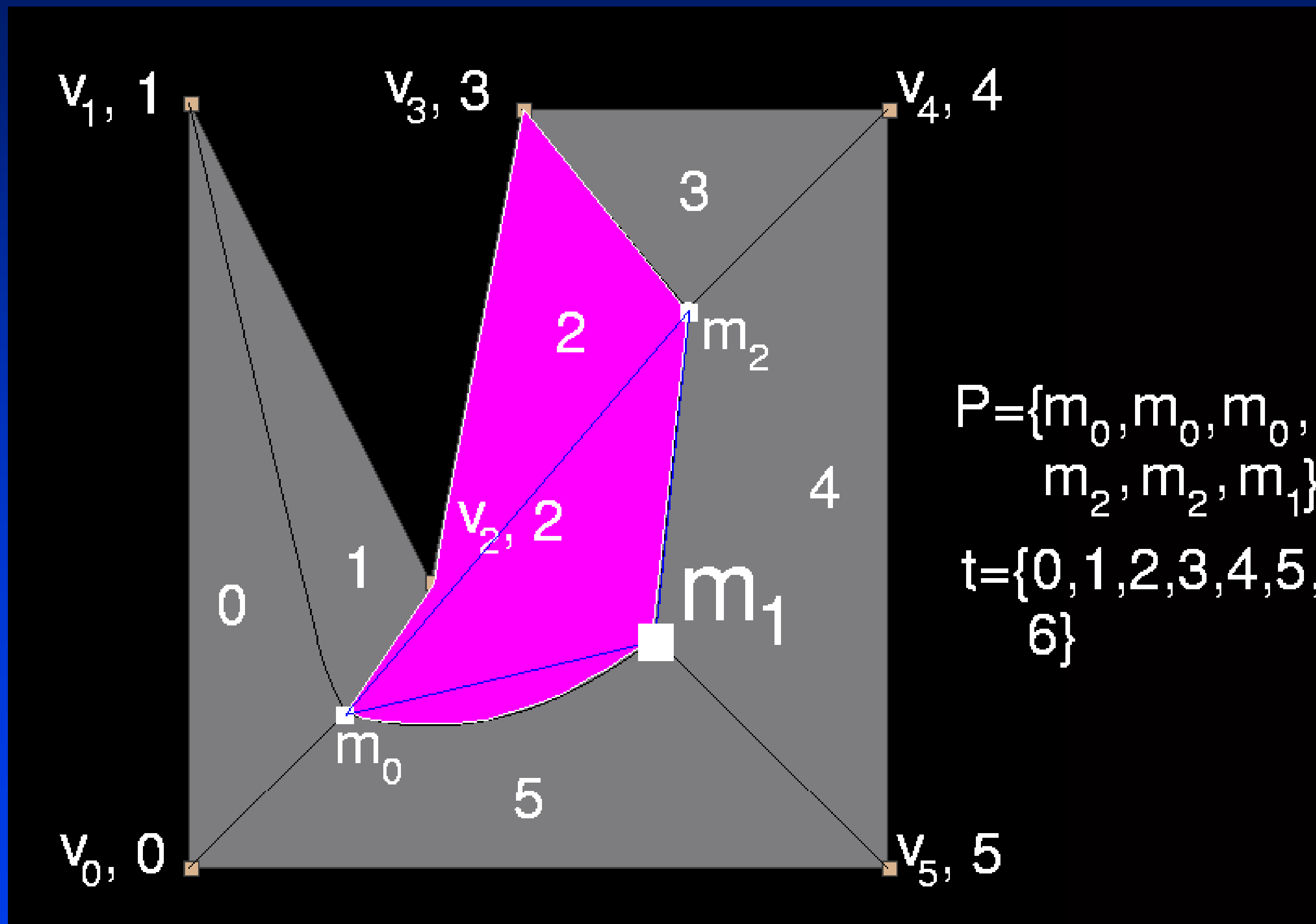
Concentric Mapping Algorithm



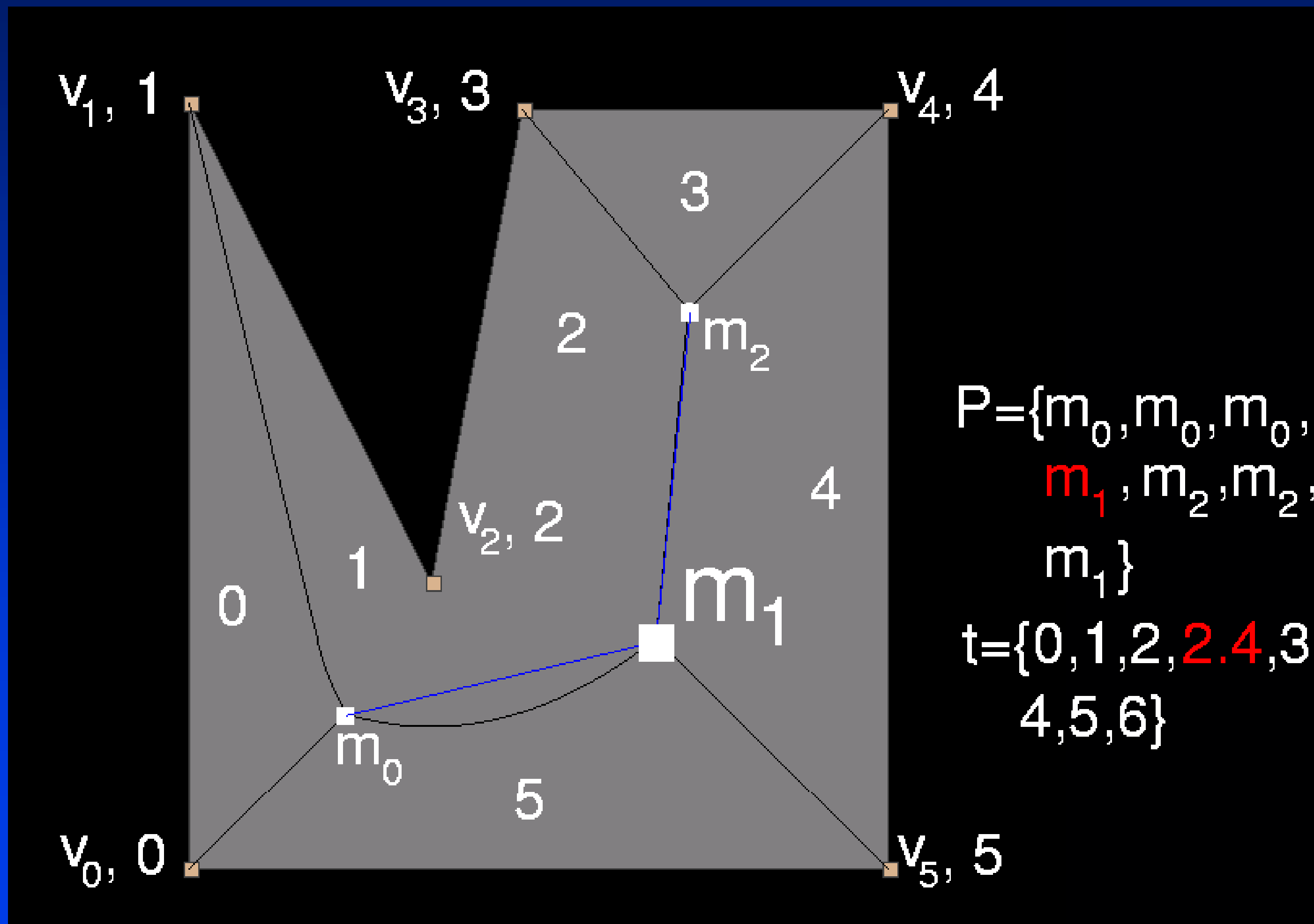
$P=\{m_0, m_0, m_0, m_2, m_2, m_1\}$

$t=\{0,1,2,3,4,5,6\}$

Concentric Mapping Algorithm

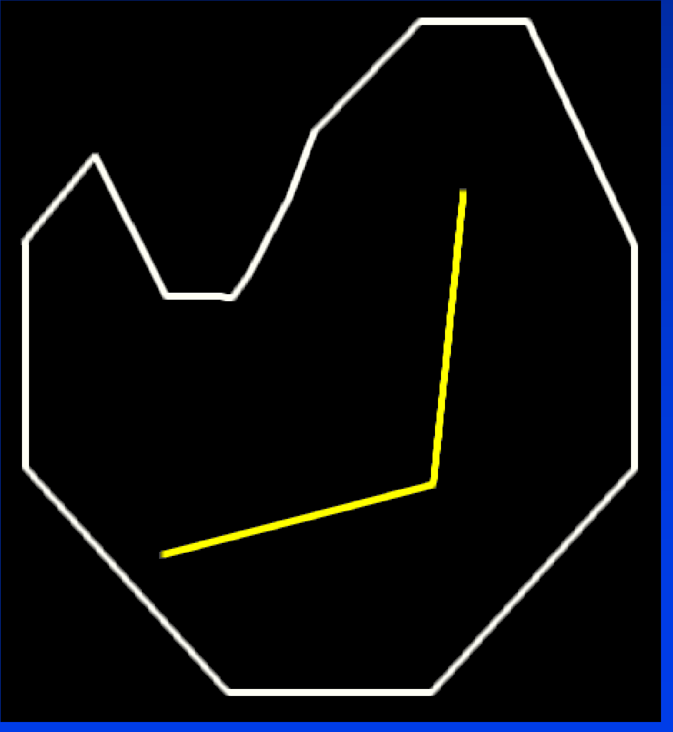


Concentric Mapping Algorithm



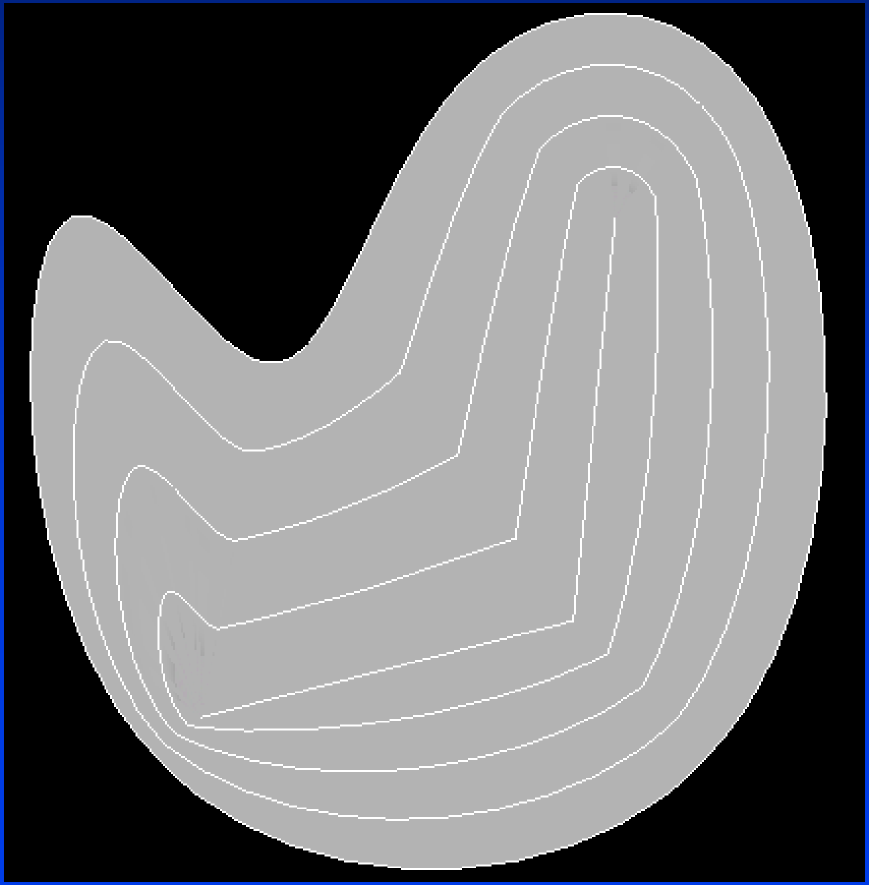
Concentric Mapping Algorithm

- 6) Degree raise the medial linear B-spline.
- 7) Refine the boundary and medial curves using the union of their knot vectors.

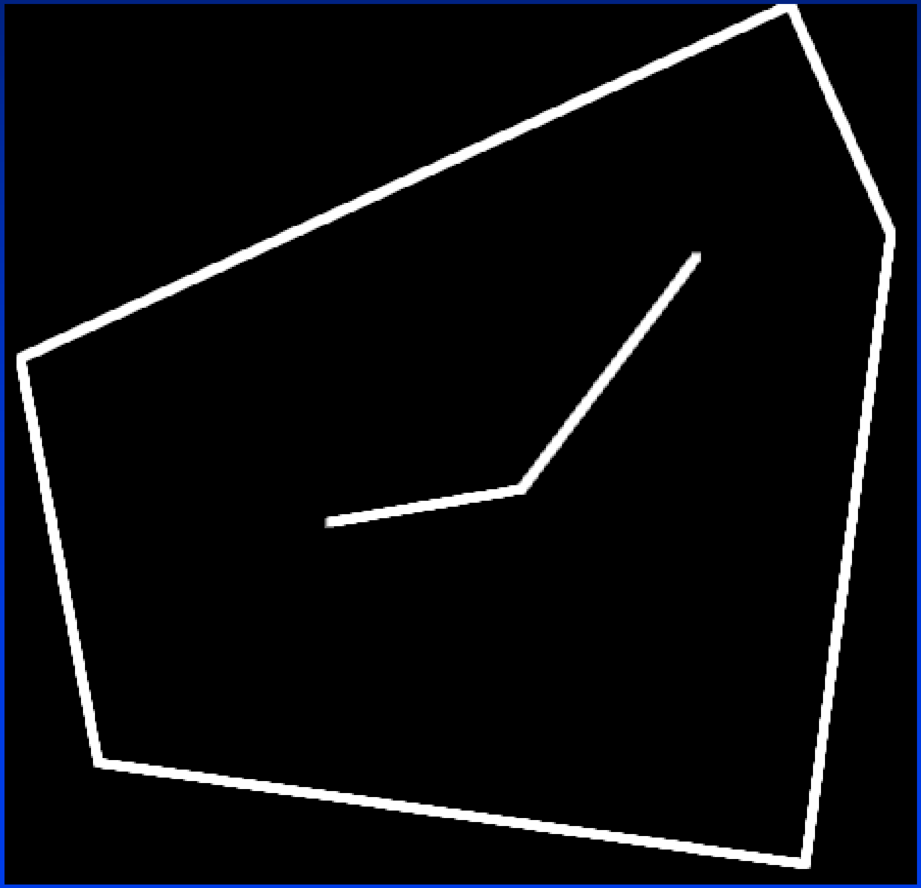


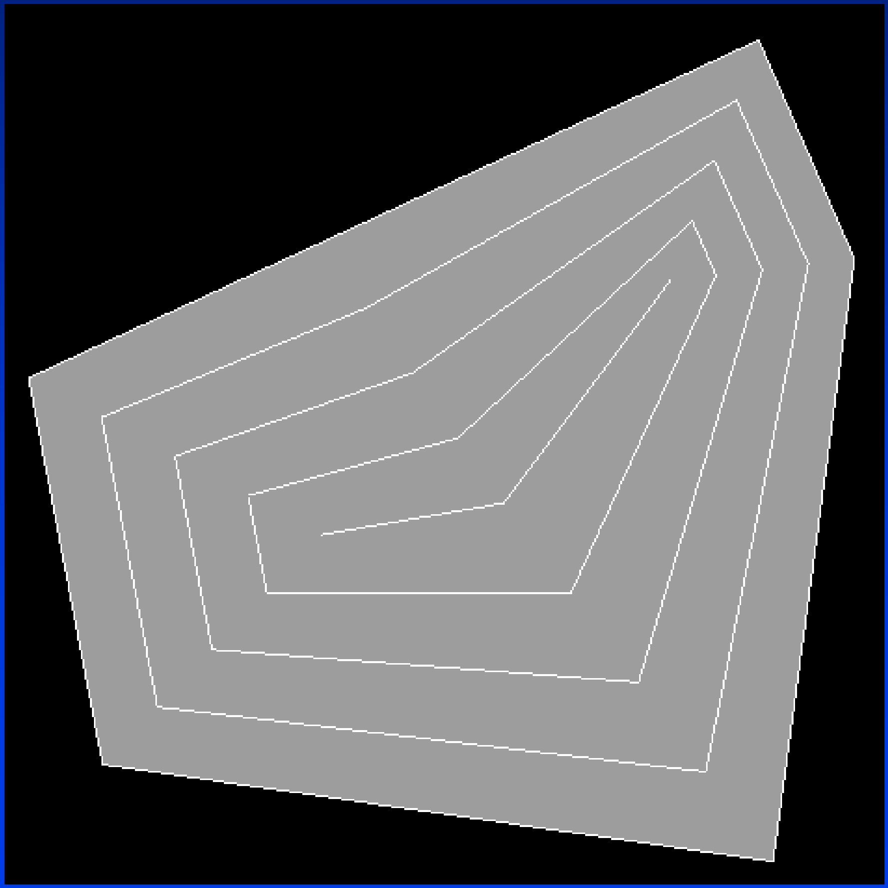
Concentric Mapping Algorithm

- 8) Form the sweep surface.

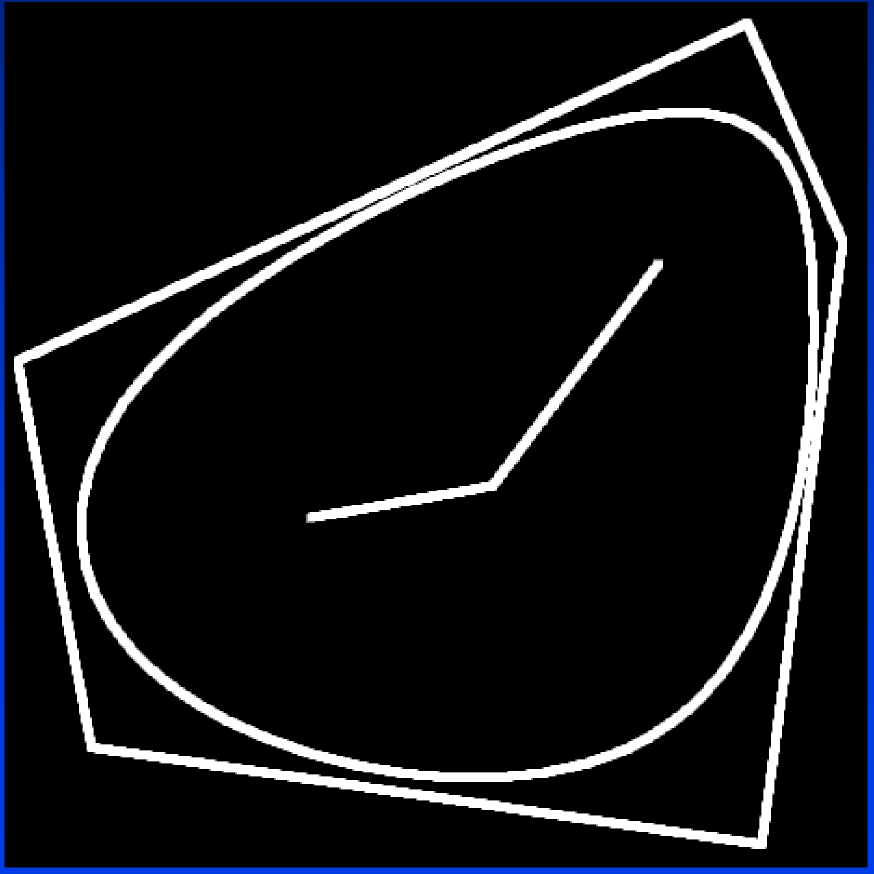


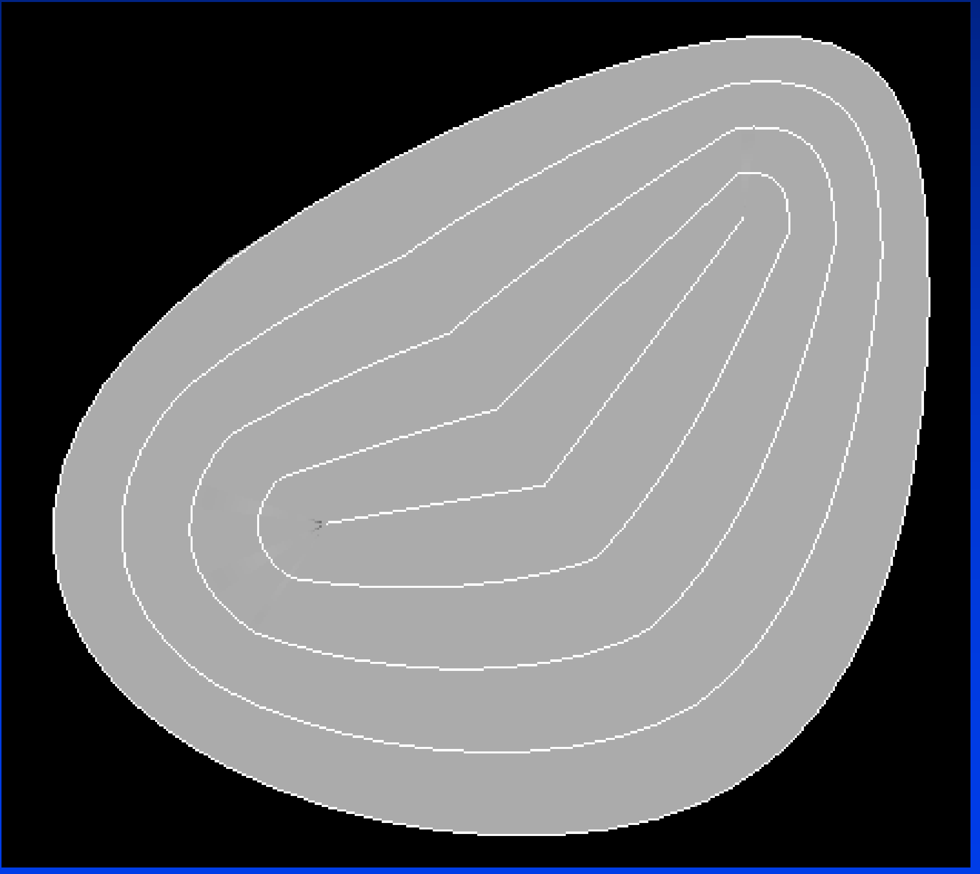
Examples: Simple Convex, Linear



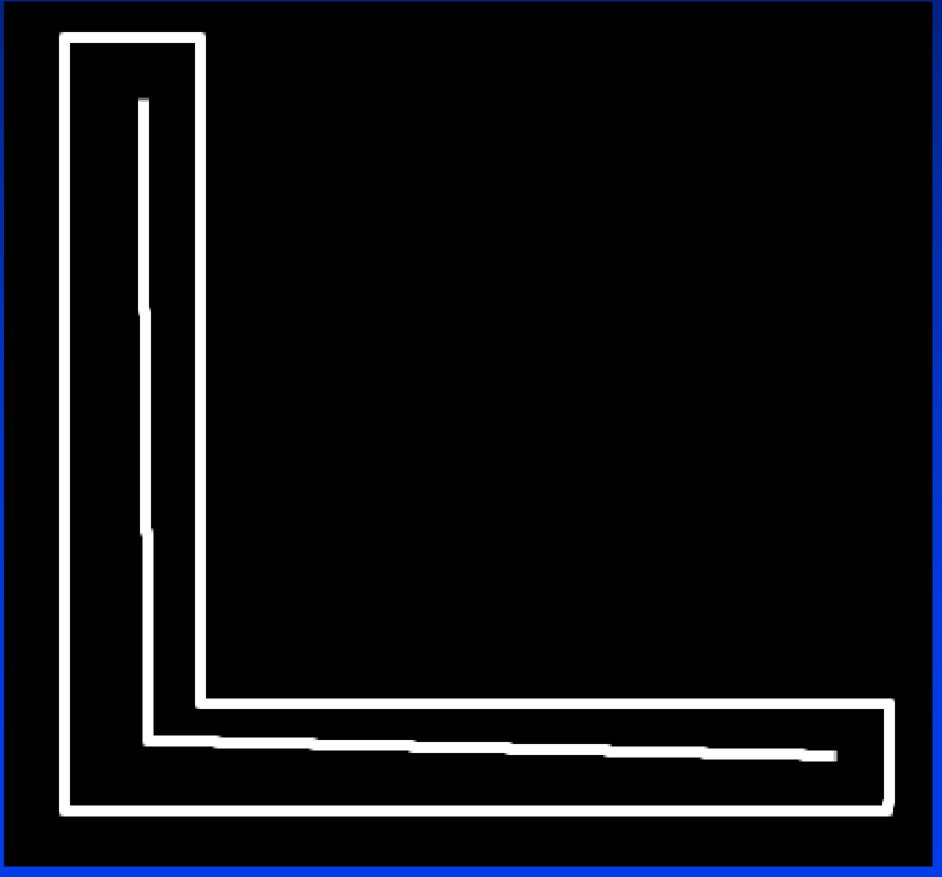


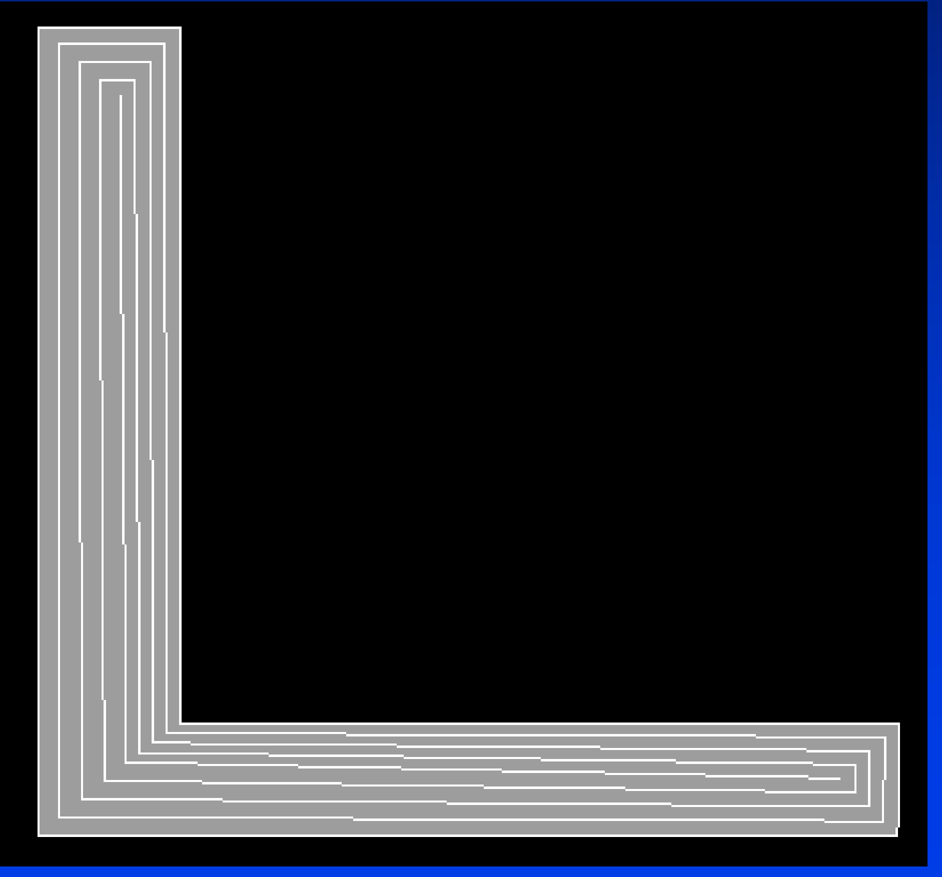
Examples: Simple Convex, Cubic



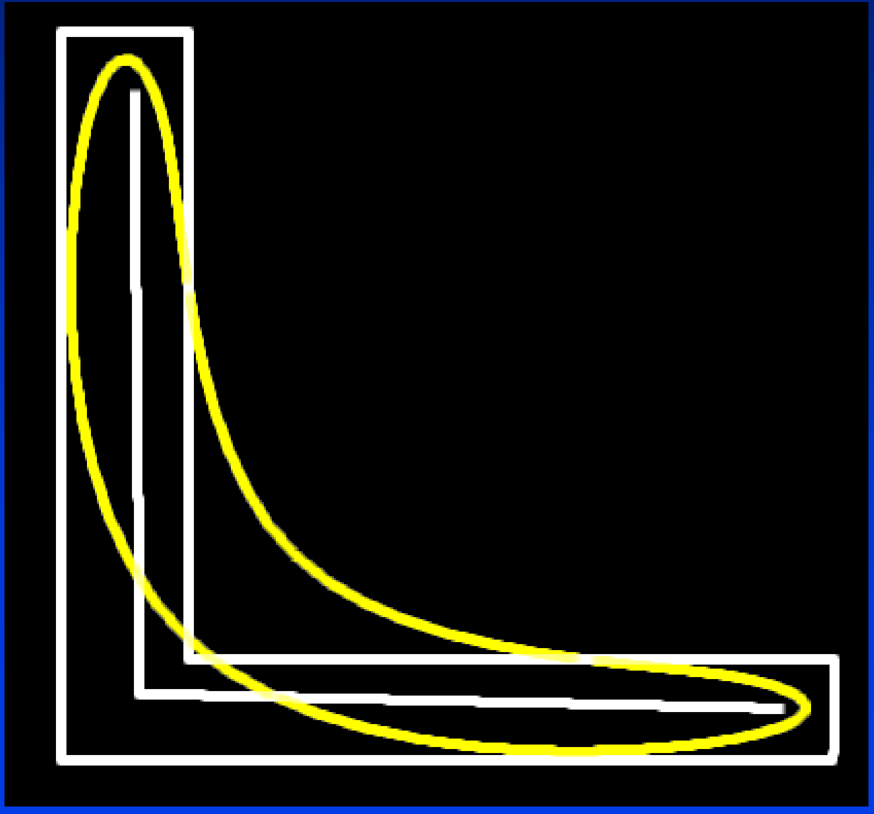


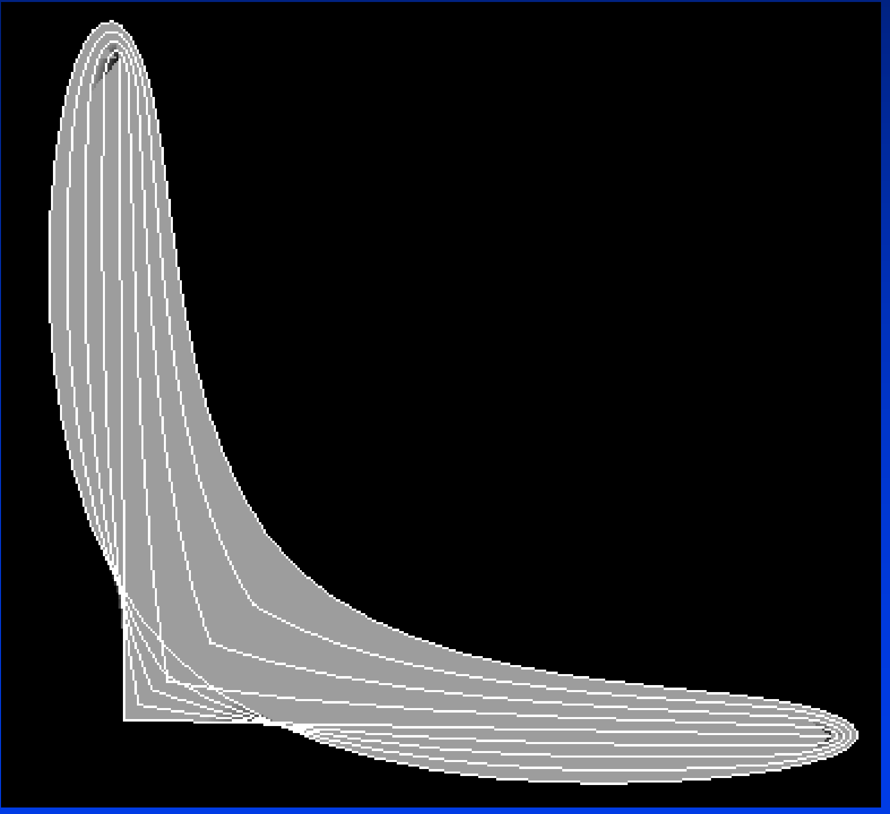
Examples: Simple Concave, Linear





Examples: Simple Concave, Cubic





Problems

However, we are mapping the new control polygon points to the medial axis of the of the old control polygon.

Medial axis may fall outside the refined control polygon.

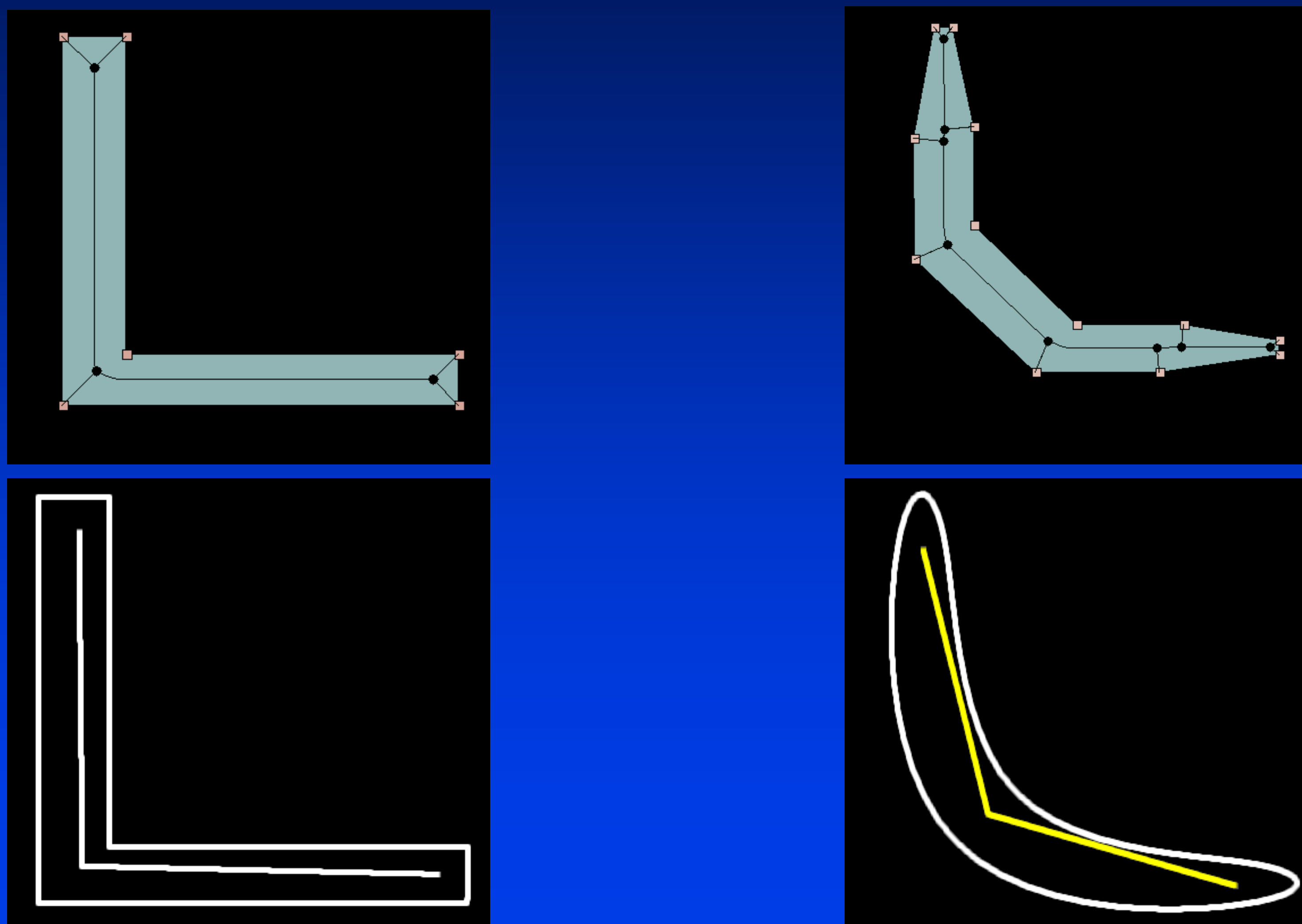
To some extent, this violates our assumption that the control polygon well approximates the curve.

Solution

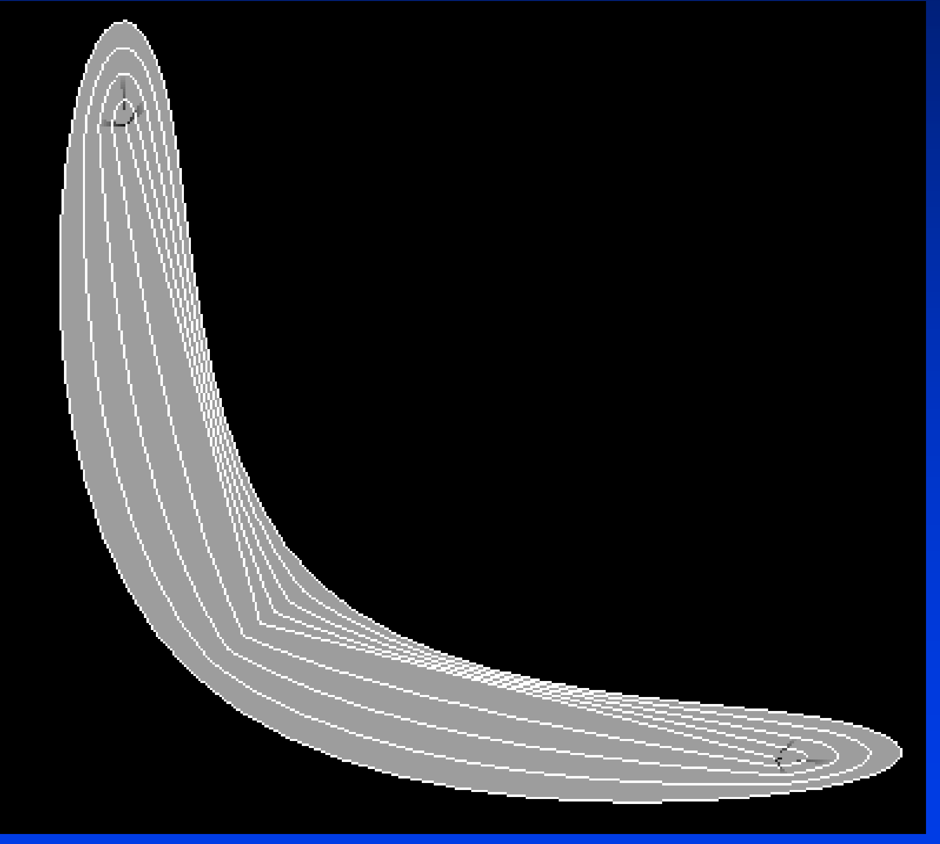
The new control polygon has a medial axis, and an associated parameterization.

Find corresponding points on the old and new medial axis, and move the old points to their new locations, keeping the topology the same.

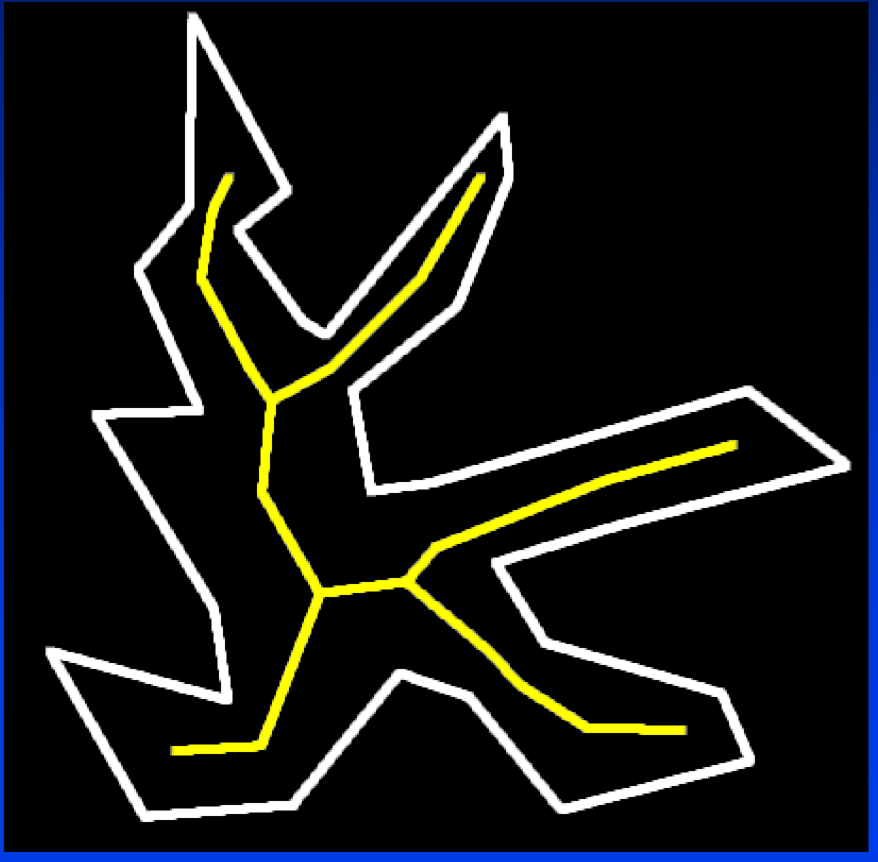
Solution

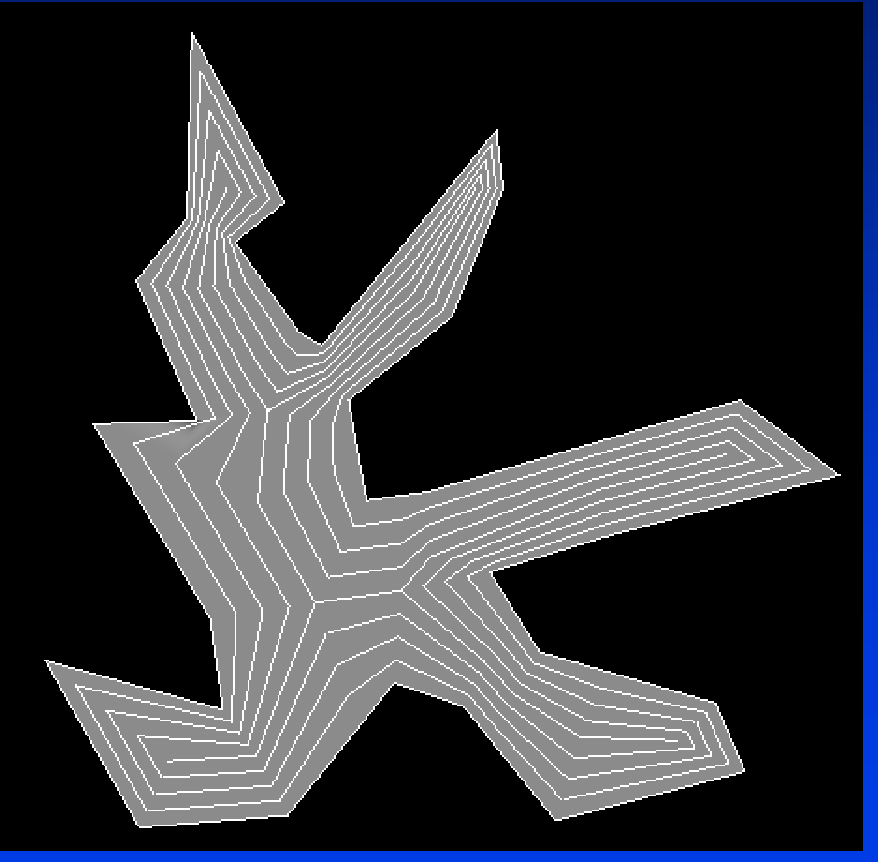


Examples: Simple Concave, Cubic

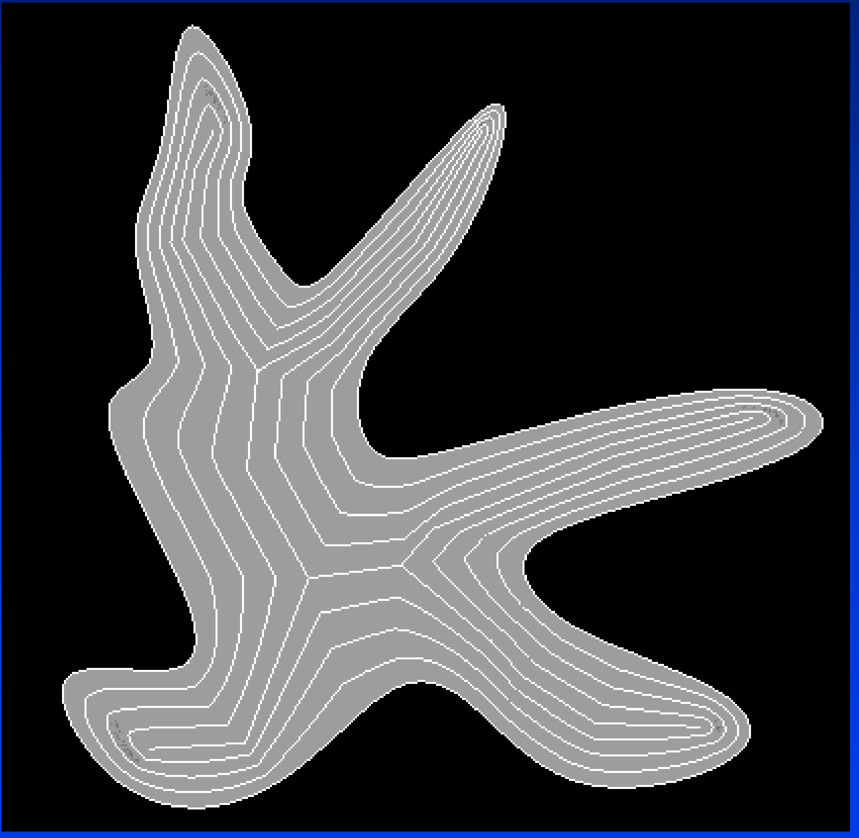


Examples: Complex, Linear

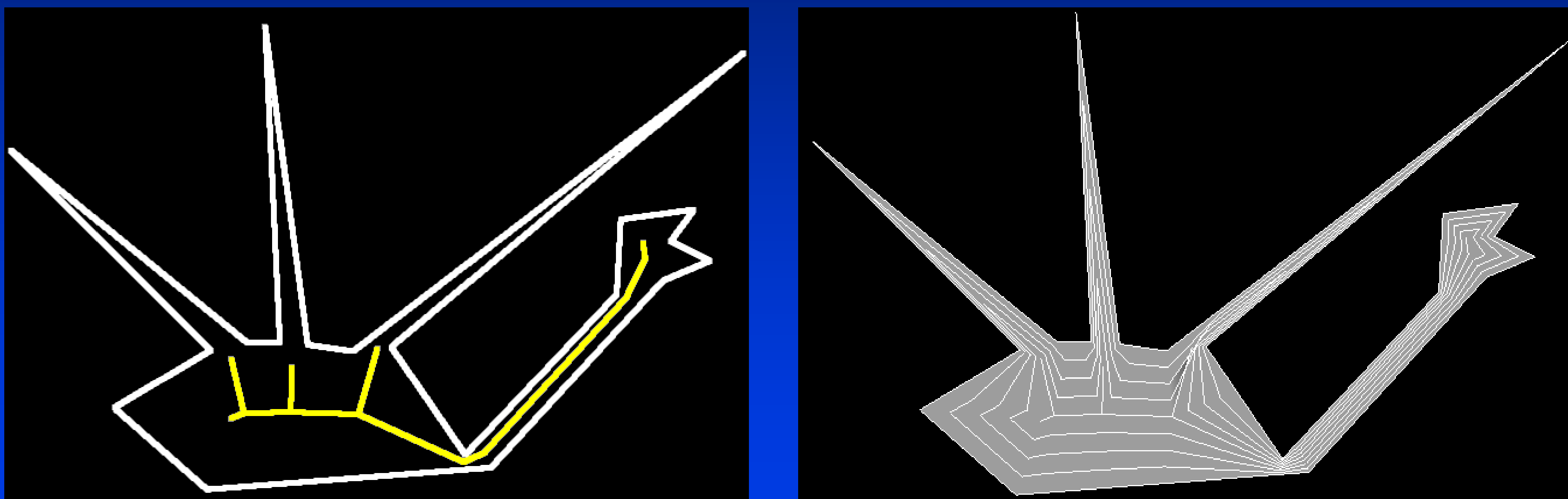




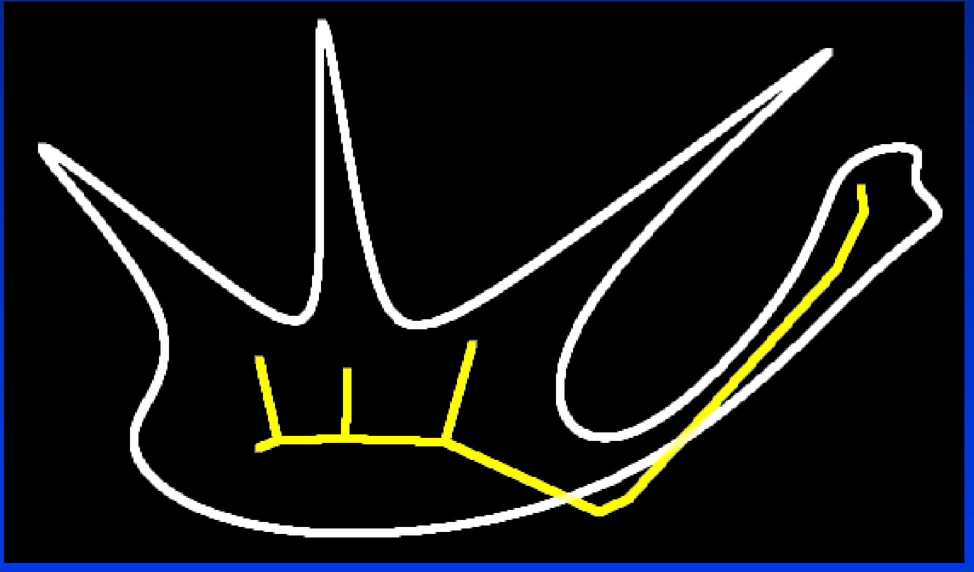
Examples: Complex, Cubic

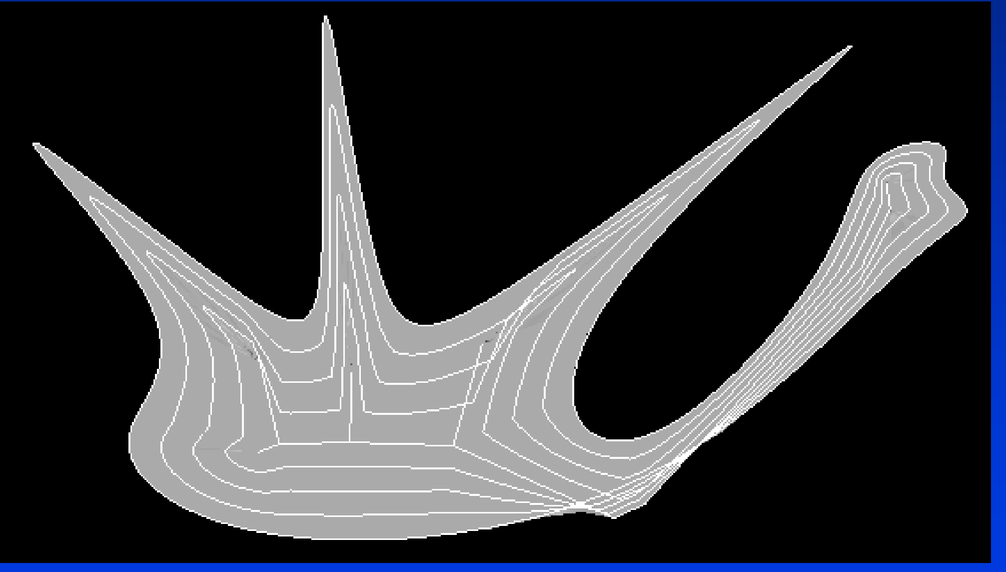


Examples: Complex, Linear

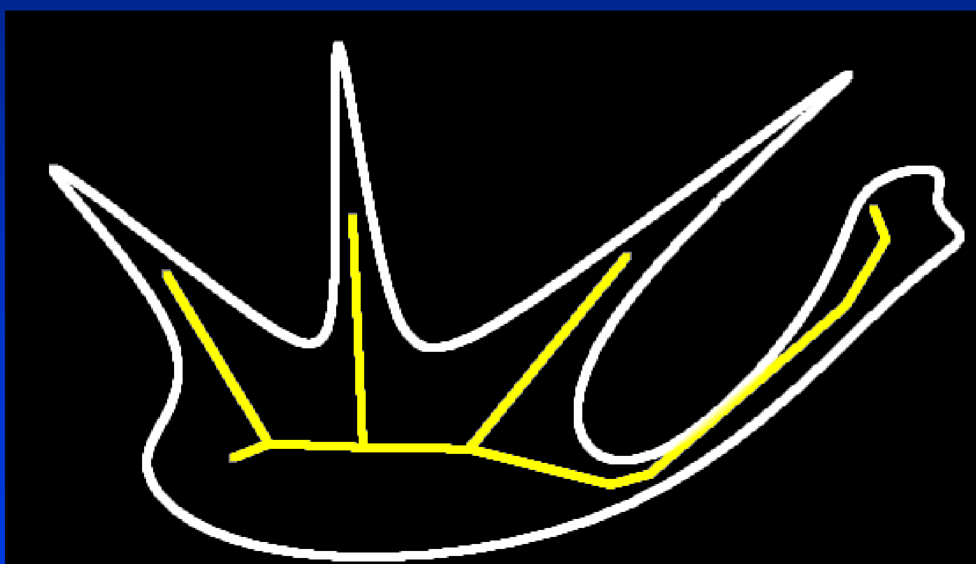


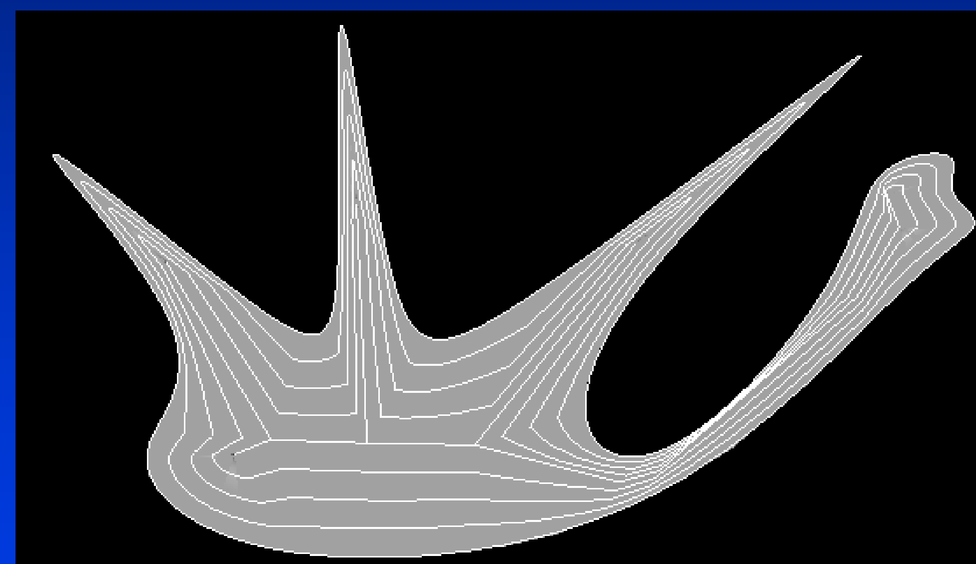
Examples: Complex, Cubic





Examples: Complex, Cubic

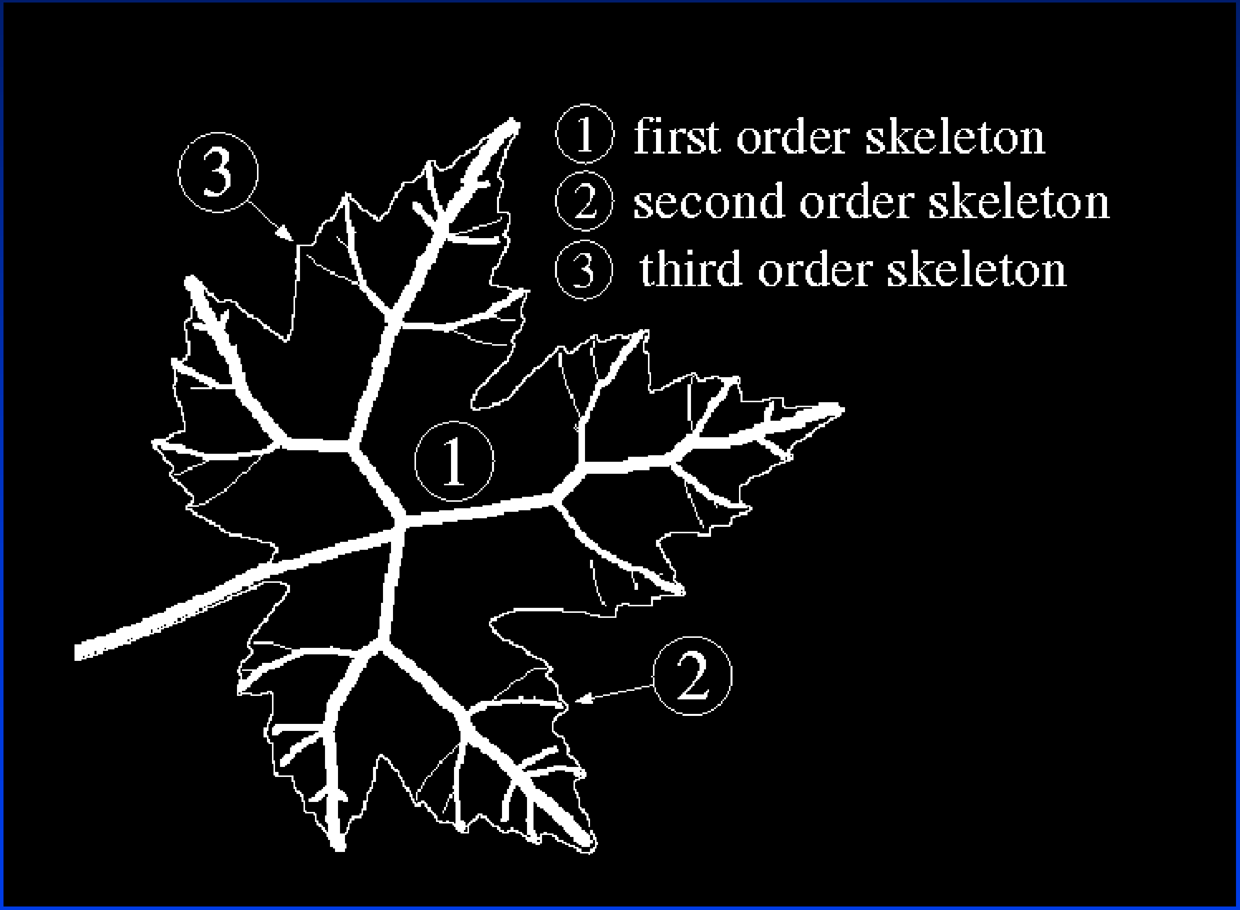




When does it not work so well?

Disparate feature sizes.

Multiscale Approaches



Ogniewicz, CVPR 1994

Future Work

Plan of research:

- Assurance of boundary containment.
- Non-planar boundary curves.
- Extensions to higher dimensions.
- Multiscale approaches.

Acknowledgements

- Amy Gooch
- Bruce Gooch
- Richard F. Riesenfeld
- Peter Shirley
- Members of the University of Utah Graphics Groups
- NSF Science and Technology Center for Computer Graphics and Scientific Visualization (EIA-89-20219)

Thank You.