

# Notes: Conditional Probability

CS 3130/ECE 3530: Probability and Statistics for Engineers

September 2, 2014

## Review of “English translation” for events:

- $A \cap B$  = “both events  $A$  and  $B$  happen”
- $A \cup B$  = “either event  $A$  or  $B$  (or both) happens”
- $A^c$  = “event  $A$  does not happen”

## Set Theory Rules: (try drawing Venn diagrams of these)

- Definition of set difference:  $A - B = A \cap B^c$  “event  $A$  happens, but  $B$  does not”

- Associative Law:

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

- Commutative Law:

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

- Distributive Law:

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

- DeMorgan’s Law:

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

## Probability Rules:

- **Inclusion-Exclusion Rule:**  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- **Complement Rule:**  $P(A^c) = 1 - P(A)$
- **Difference Rule:**  $P(A - B) = P(A) - P(A \cap B)$

Exercise: Try deriving these rules from the definition of a probability function. Draw a Venn diagram to convince yourself they work.

## Conditional Probability:

$P(A|B)$  = “the probability of event  $A$  given that we know  $B$  happened”

$$\text{Formula: } P(A|B) = P(A \cap B)/P(B)$$

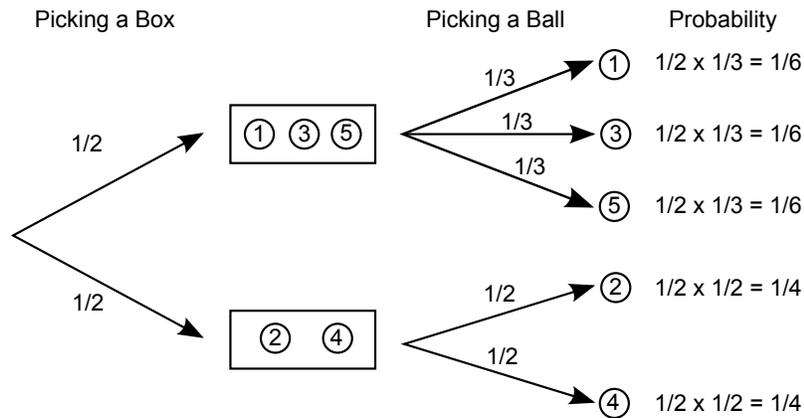
## Multiplication Rule:

$$P(A \cap B) = P(A|B)P(B)$$

**Tree diagrams** to compute “two stage” probabilities ( $B$  = first stage,  $A$  = second stage):

1. First branch computes probability of first stage:  $P(B)$
2. Second branch computes probability of second stage, given the first:  $P(A|B)$
3. Multiply probabilities along a path to get final probabilities  $P(A \cap B)$

Example: You are given two boxes with balls numbered 1 - 5. One box contains balls 1, 3, 5, and the other contains balls 2 and 4. You first pick a box at random, then pick a ball from that box at random. What is the probability that you pick a 2?



**Sampling without replacement:**

I have a box with 10 red balls and 10 green balls. I draw 2 balls from the box without replacing them. What is the probability that I get 2 red balls?

Let  $R1$  = “first ball red” and  $R2$  = “second ball red” and use product rule:

$$P(R1 \cap R2) = P(R1)P(R2|R1) = \frac{1}{2} \times \frac{9}{19} = \frac{9}{38} \approx 0.24$$

If I draw 3 balls without replacement, what is the probability that they are all red?

$$\begin{aligned}
 P(R1 \cap R2 \cap R3) &= P(R1 \cap R2)P(R3|R1 \cap R2) && \text{Multiplication rule for } (R1 \cap R2) \cap R3 \\
 &= P(R1)P(R2|R1)P(R3|R1 \cap R2) && \text{Multiplication rule for } R1 \cap R2 \\
 &= \frac{1}{2} \times \frac{9}{19} \times \frac{8}{18} = \frac{18}{171} \approx 0.11
 \end{aligned}$$

In-Class Problem: Exercise 3.2a